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Concept of Constructing a Technological Model for Solving Semi-structured Problems on the Basis of Set Theory

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Abstract. This article discusses one of the approaches to solving semi-structured problems using the apparatus of set theory. The problem is defined as a system of inhomogeneous elements, the structural analysis of which allows identifying incorrect elements. The further solution is presented in two stages. At the first stage, a block diagram of the solution of the problem is developed with the last level of the hierarchy detailed to the maximum. At the second stage, the description of the structural blocks is given in symbols of set theory. As an example, the problem of processing trajectory measurements is considered. The incorrect blocks of the problem are pinpointed and the technology of the general solution is given taking into account the representation of these blocks in symbols of set theory.

Keywords: systems approach, structuring, system, sets, model

Introduction

One of the problems related to the field of systems analysis is the search for a solution to the problem of semi-structured problems [2, 4]. Solving this problem is complicated by the absence of boundaries that would establish the belonging of tasks to the semi-structured type, as well as by the lack of specific criteria for determining the degree of structuredness. Allowing that semi-structured problems are located in the range from structured to non-structured problems, it can be assumed that the solution–result may vary from quantitative to qualitative and, in some cases, it may not be found.

The solution of the problem, virtually, is reduced to the identification of incorrect elements in the software and algorithmic shell, to operation monitoring and, in the case of failure, to transferring the solution to another chain of the algorithm.

The present paper describes one of the possible approaches to solving a problem consisting in the structural analysis of the algorithm and the subsequent representation of the incorrect elements of the solution in the form of set theory procedures.

The term “problem” (or task) is understood as a system consisting of elements, which ensure the reception of an answer to a posed question.

Preliminary notes

There are three ways of representing a solution to a problem: an algorithm, a block diagram and a verbal description.

1. Algorithm. It is used for formalized problems that a have a mathematical solution.

2. Block diagram. It is implemented for displaying the procedure of solving problems in the form of a hierarchical sequence of interrelated functional blocks.

Structural analysis of the block diagram allows you to mark the blocks capable of leading to an incorrect decision; however, it is impossible to determine the specific function or parameter, which is the source of this situation.

3. Verbal description. It is acceptable for the first and second variants of problem structuredness mentioned above but it is only presents a qualitative picture of the solution without the possibility to formally adjust the process.

In the second and third types of problem representation, separate elements may be of a non-formal nature, which makes it difficult to construct programs using modern programming languages and, consequently, hinders the process of automating the decision-making process.

The essence of the method lies in the use of logical conditions for determining the truth (or falsity) of the set of block elements by matching one of its parameters to a specified value. This permits to establish relationships between blocks of the solution of the problem, which is presented in structural or verbal forms.

The problem solution is divided into two stages. At the first stage, a block diagram (flowchart) of the problem solution is developed with maximum detailing of the last level of the hierarchy. At the second stage, the structural blocks are described using symbols of set theory.

Based on a sufficient number of works dedicated to the study of problem structuredness and their solution [2-6], it can be said that problems are divided by the degree of structuredness into: structured (SP), semi-structured (SSP) and non-structured tasks (NSP).

The quantitative and/or qualitative **definiteness** of the elements of the problem is used as a criterion for classifying elements of the problem. Another criterion for dividing problematic situations into structured, semi-structured and non-structured is the extent the algorithm of their solution is known [2]. Considering the abovementioned, the following descriptions can be given to each problem type.

SP: are characterized by the presence of a solution algorithm based on mathematical relationships as well as by a quantitative result.

SSP: are known for their qualitative relationships between elements of the problem, information about part of the elements may be absent.

NSP: are characterized by the absence of a mathematical solution algorithm and by the partial absence of conditions and source data.

It is appropriate to add quasi-structured problems (QSP) to the given list of problem types determined by the level of problem structuredness. These problems are characterized by the presence of quantitative and qualitative constituent elements with a probability of failure of the latter during the problem resolution process but with a possibility of finding a quantitative or qualitative solution. This problem type is situated between SP and SSP.

Graphically, this distribution of tasks by the degree of structuredness is given in Figure 1.

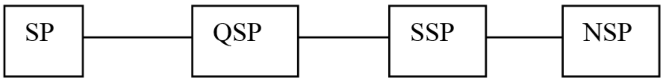

Fig.1. Distribution of tasks according to degree of structuredness.

A structured problem was taken as a baseline variant (rating) for conducting the study.

The level of structuredness of a task was determined by the ratio of correct and incorrect properties inherent in its constituent blocks.

Depending on the functional load of the elements comprising the solution, there can be three forms of representing the solution:

- software (SW; program-algorithmic);
- information computing (IC);
- hardware/technology (HT).

Each form of representing a task is comprised of the following elements:

- software and analytical (SA): statement + algorithm + program;
- information computing element: SA + database (DB) and archives;
- hardware and technology elements: IC + computer + data path for inputting measurements into DB.

Regardless of belonging to one or another form, a problem has the same set of elements given below.

Elements of a problem

Functional parts of the problem

1. method — statement, algorithm;
2. software — operating system (OS), programming language, program codes, standard subroutine library (SSL);
3. calculation — computing procedure (processor: type, frequency), random access memory.

Data part:

1) DB, archives, data exchange (communication);
2) interface — input and output data, control of intermediate computations.

Stages of solving the problem:

1) development of solution structure, method and algorithm;
2) program development and performing calculations.

Ways to solve the problem:

1) formal solution (algorithm and software development);
2) informal solution (search for an alternative solution);
3) semi-structured solution (in the case of a failure possibility) – divided into a “quality” or “quantity” solution, depending on the ratio of formalized and non-formalized elements of the problem as well as on the real conditions of the solution.

Data description formats:

1) quantitative;
2) qualitative;
3) set-format (sets and operations with them).

Description of problem solution model:

1) mathematical algorithm;
2) architecture (block diagram);
3) description of the sequence of the stages of solving the problem;
4) technological model in the form of a description of problem structural blocks and their connections using set symbols.

As an example, the article considers a solution to the problem of processing trajectory measurements of the ballistic-navigation support (BNS) with respect to the software (program-algorithmic, SW) form.

The majority of tasks concerning measurement processing are structured tasks. Yet, in the presence of three incorrect elements in the structure of such a problem, its type may change.

The specific features of such problems are:

— a large volume of measurement data saved in the source data (SD);

— the presence of a restriction on the quantity and quality of measurements in the algorithm;

— the main measurement processing method is statistical (least-squares method (ordinary least squares), OLS, Kalman filter);

— the use of a mathematical model in the algorithm for every measurement type (range, phase, Doppler frequency and etc.); and

— the implementation of the following mathematical functions: calculation of partial derivatives, composition and solution of systems of linear equations, formulas of calculated values of measured parameters.

Given below are the results of analyzing a problem for determining the correctness of solution blocks.

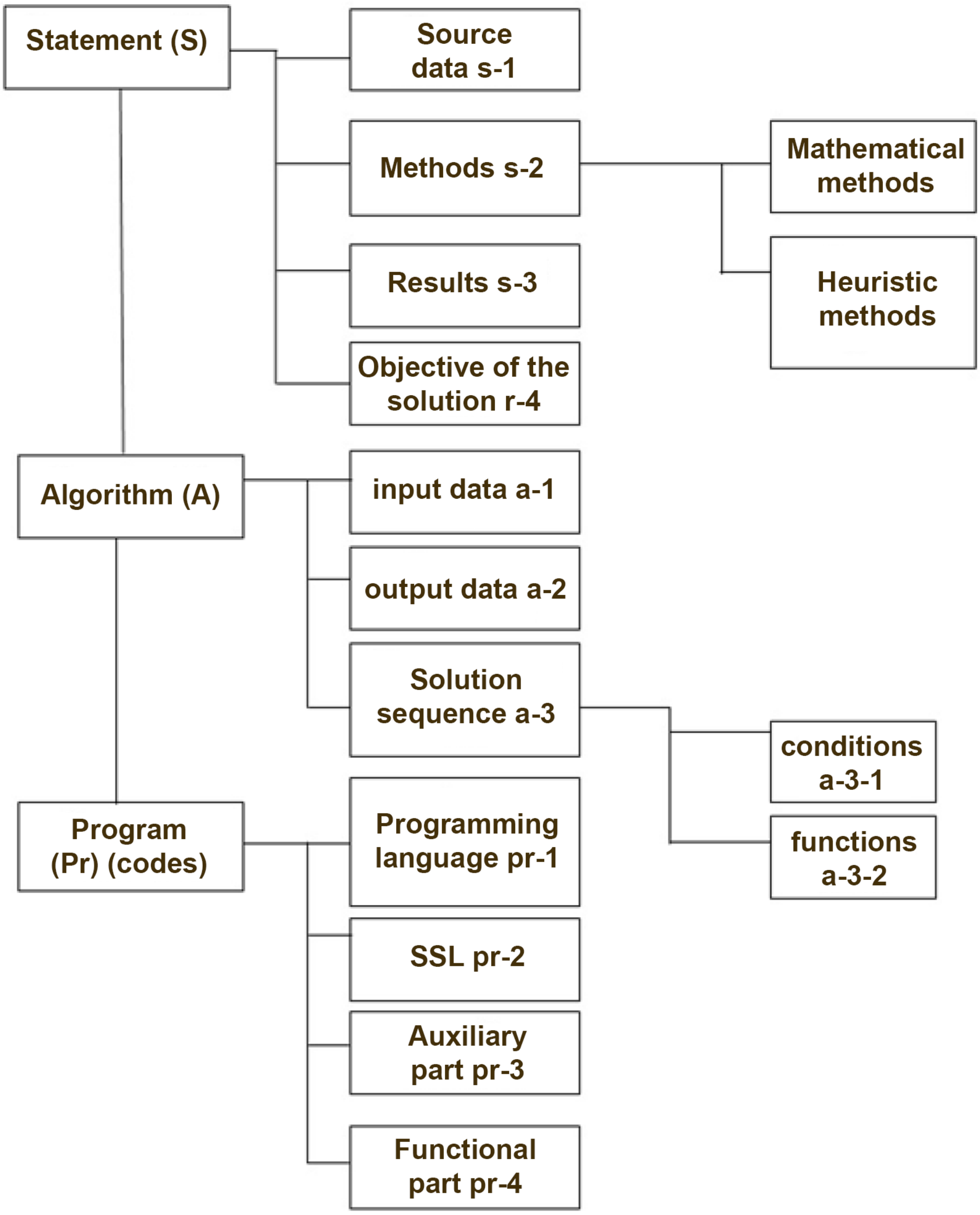

Fig.2. Block diagram of problem in SW form.

Problem structure

First, we will set forth a block diagram of a problem in SW form (Figure 2).

The auxiliary part includes:

- source data processing and generation
- access to DB
- access to archives
- access to SSL
- loop organization (looping)
- conditional and unconditional branching between program parts, and
- generation of output data.

The functional part includes:

- a mathematical model of the considered physical process;
- iterative calculation methods
- control of correspondence of mathematical formulas to the physical process being described
- methods of interpolation and extrapolation
- numerical methods for solving differential equations.

Based on the distribution of problems according to the degree of structuredness and to the block diagram (Figure 2), Table 1 gives data on the correctness (+) and incorrectness (-) of the blocks distributed across problems of varying degrees of structuredness and its constituent parts.

Table 1. Distribution of problems according to the degree of element structuredness

Parts of the problem	Symbol for blocks of problem parts	Types of problems in accordance to their structuredness			
		SP	QSP	SSP	NSP
Statement	s1	+	+	-	-
	s2	+	+	+	-
	s3	+	+	-	-
	s4	+	+	+	-
Algorithm	a1	+	+	-	-
	a2	+	+	-	-
	a3-1	+	-	-	-
	a3-2	+	-	-	-
Program	pr1	+	+	+	+
	pr2	+	+	+	+
	pr3	+	+	+	+
	pr4	+	-	-	-

Table 2. Dependence of solution on problem type

Type	Solution		
Problem type	Quantitative	Qualitative	0
SP	+	-	-
QSP	+	+	-
SSP	-	+	+
NSP	-	-	+

Table 2 gives classification data on the distribution of types of problem solutions depending on the degree of problem structuredness, where "+" is the correctness of the solution of the corresponding problem type; and "-" is the problem solution incorrectness (the absence or the non-compliance with the conditions of normal solution functioning). A quantitative solution is a result in numerical format, a qualitative solution is a result in verbal form; 0 – is the lack of options for solution continuation: obtained result does not correspond to mission in terms of physical sense.

Data from Tables 1 and 2 allow you to define the degree of structuredness of the problem, determine its place in the proposed classification, as well as refine the incorrect blocks in the main elements of the problem.

The received information makes it possible to preselect the appropriate solution method with account for the possible incorrectness of separate parts of the problem.

Block diagram of the problem

Given below is a block diagram (flowchart) of the functional blocks of the problem of measurement processing, where blocks with a probability of an incorrect solution are marked (in yellow) (Fig. 3).

The apparatus of set theory is proposed to be used for the transition from a structural display of problem elements and their relationships to a formalized form.

In this case, each block (which is a system) can be identified as a set of heterogeneous elements. The relations between problem elements (Figure 3) can be represented in the form of a corresponding relationship between the parameters of sets [7].

The top level of structural blocks:

p = s;

S = std SD = s1

A = a1

R =

Let us represent the conditions for the selected structural blocks in the form of the following relations.

Initial data:

s1 = { φ , **γ**};

s1 = {$_i$, measurement limitation for $_i$<;

s1 = {k , limit on the number of measurements per session.

Formulas

a1 = { , establishing the correctness of the formulas.

Matrices:

, condition for using matrices in the algorithm

where Mt are the transition matrices of coordinate systems;

Ms are the matrices for solving systems of linear equations.

Composition of sought data:

p1 = {Q, n},

Q – is the form (type) of data representation (for example, orbit parameters are osculating elements);

n – is the number of data (for example, a kinematic vector of state – 6 values)

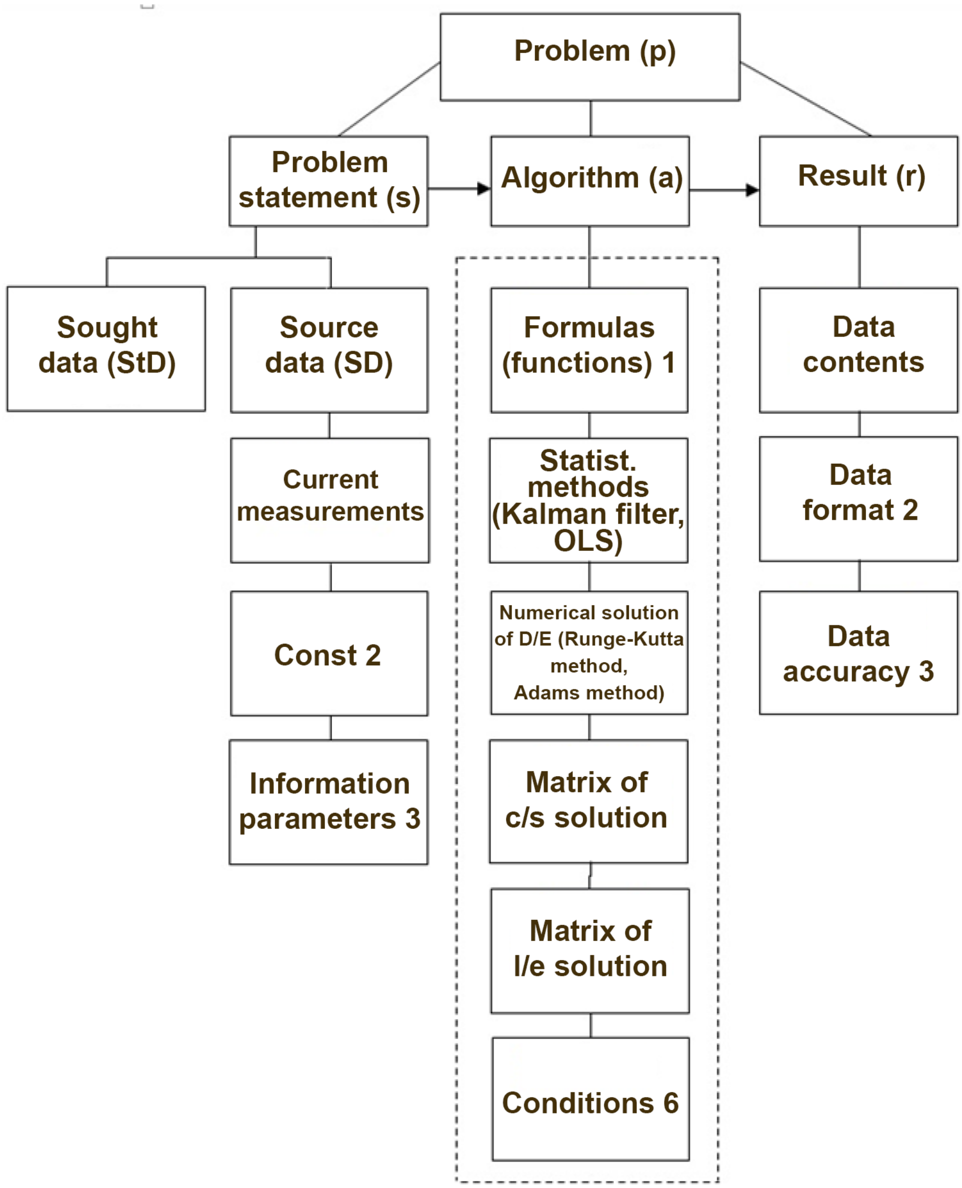

Fig. 3. Diagram of functional blocks of measurement processing: d/e – differential equation, l/e – linear equation, c/s – coordinate system.

$Q = \{Q_i : Q_j\}$, the condition for choosing the type of result corresponding to the given one.

$Q_i = \{s = n\}$, match condition of the number of data.

Accuracy of sought data:

s3 = {σ , v}, root-mean-square deviation (RMSD) and the number of significant digits after the decimal point.

s3 = { σ_i: σ_n}, condition for checking the result for accuracy by RMSD.

s3 = { v_i : v_n}, condition for checking the result by the number of digits after the decimal point.

s = { s , condition for choosing result by the fulfillment of condition p3.

The first stage of solving the problem

As an example of implementing the proposed method, we will consider the problem of refining the initial conditions of spacecraft (SC) motion using trajectory measurements. The problem solution is divided into two stages: at the first stage, the solution will be given in form of a flowchart (block diagram) (Figure 4); at the second step, it will be given in the form of an algorithm in the language of set theory.

1st stage of solution

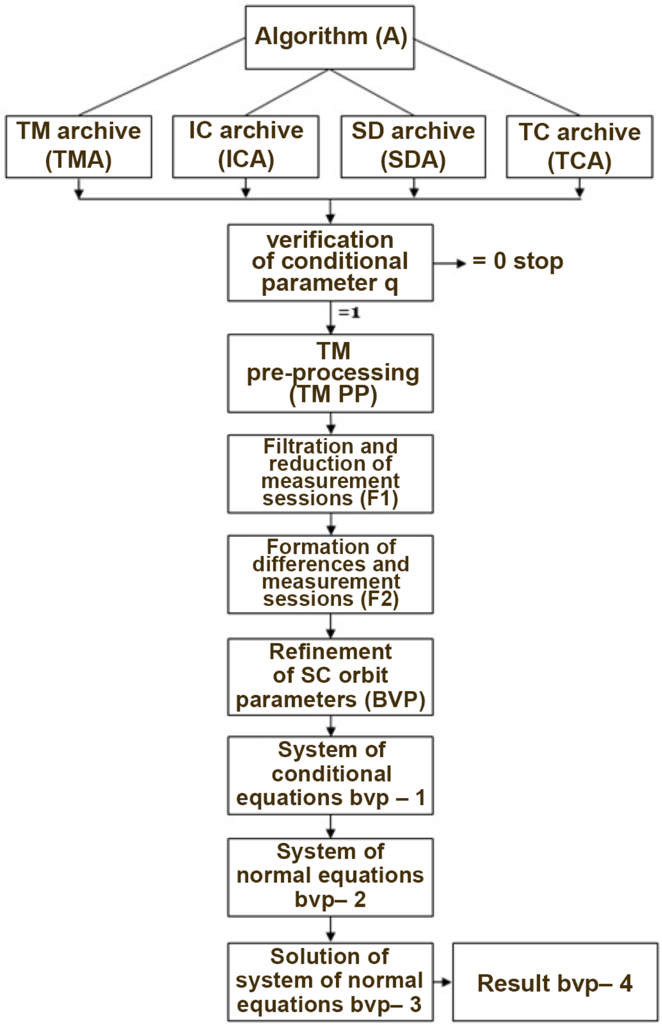

Fig. 4. Flowchart (block diagram) of solution for problem of refining initial conditions.

We will give explanatory notes to Figure 4.

TM archive – is the file archive of trajectory measurements. The principal format for TM are Rinex-files.

IC archive – is the file archive of SC initial conditions obtained at previous BNS-session. The main form: date, time and parameters of the kinematic state vector in the Greenwich coordinate system (GCS);

SD archive – is the archive of source data. The main contents: global constants of the parameters of Earth's gravitational field, Earth rotation parameters, etc.

TC archive – archive of technical characteristics of the spacecraft. The main contents: SC number, SC type, time delay of signal relay, nominal values of carrier values of navigation radio signals from onboard generators;

BVP – is the boundary value problem - the generally accepted name of the task of refining the ICs. Below, the blocks that make up the BVP belong to the main stages of processing measurements with the help of OLS;

Result – are the refined values of the components of the SC kinematic vector of state in GCS at the start of measurement.

The second stage of solving a problem

Every block of the aforementioned flowchart (block diagram) is a set with heterogeneous elements. Therefore, the transition to the form of a solution in symbols of set theory is the first approximation to the final form of the algorithm for the further development of the program. This example allows us to talk about the fundamental possibility of such an approach to the solution of semi-structured problems.

A = {TMA ;

TMA = ICA = SDA = TCA=

Conditions of the correctness (validity) of data in the archives are checked.

Let us introduce conditional parameter *q*. If the correctness conditions *q* of these archives are met, then *q* takes on the value of 1, otherwise – 0.

TMA = {}

TMA = {$_i$, limitation on measurements for $_i$<, where **φi** are measurements.

TMA = {k, limitation on the number of measurements in session N.

ICA = {, limit on the value of RMSD.

SDA = {$v_i v_n$}, limitation of data by the number of digits after the decimal point.

TCA = { correspondence of TCs to nominal values.

After checking each archive, depending on the outcome of the procedure, the conditional parameter *q* was assigned a corresponding value (0 or 1).

The value of the conditional parameter is checked.

If q = {= 1}, then the action goes to the block "TM preliminary processing (TM PP)";

If q = {= 0}, the solution comes to a **stop**.

TM PP = {f1 ;

where f1 = {y1,y2},

y1 = {$_i$, limitation of $_i$< measurements.

y2 = Δt = {(t_i- t_{i-1}) : δt}, condition for reduction of measurement sessions;

t_i , t_{i-1} – are the start times of two adjacent measurements, δt is the given (specified) time interval.

f2 = {y3,y4},

y3 = {Di = D_3}, limitation on the length of baselines *Di*, (D_3 is the given length).

y4 = {k , limitation on the number of measurements in session *N*.

We shall consider the block for refining the parameters of the orbit (BVP).

BVP = {bvp1 bvp;

The general form of the conditional equation $r_{meas} - r_{est} = \Delta r$, the number of equations is equal to the number of measurements *s*.

BVP1 = {n = s};

The system of normal equations can be represented as follows

$$a_{11}P1 + a_{12}P2 + \ldots + a_{1k}P_n = A_{11}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{k1}P1 + a_{k2}P2 + \ldots + a_{kk}P_n = A_{kk}$$

The number of equations in the system should be equal to the number of parameters being refined.

bvp2 = {n : k};

The general form of the solution of a system of normal equations:

[p] = B, B = H^{-1} – is the product of the inverse matrix of partial derivatives for the measured values with respect to the sought parameters by the correlation matrix of the measurement vector.

bvp3 = [p] =, matrix H must not be equal to 0, otherwise we will have division by 0.

The result of the solution of the BVP is a state vector refined by the corrections received by solving a system of normal equations.

$= _0 + p$

where – is the refined state vector of the SC;

$_0$ – is the initial value of the state vector of the SC;

p – is the vector of corrections as a result of solving the BVP.

: }, is the correspondence of the obtained solution to the given (set) type (format, accuracy).

The presented two-step technological model for solving semi-structured problems cannot pretend to be a complete algorithm but is can greatly facilitate the search for a solution due to the overcoming of situations that arise from the incorrectness of separate elements in the blocks of the problem in a formalized manner.

Conclusion

The following conclusions can be made based on the materials in the present article.

1. A technological model for the solution of semi-structured problems has been developed based on set theory.

2. The technological model is implemented by sequentially performing the following steps:

- development of a block diagram (flowchart) of the problem solution;
- allocation of incorrect (invalid) blocks;
- representation of the conditions of incorrectness in symbols of set theory.

3. The creation of a complete algorithm for the solution of semi-structured problems is possible by the introduction of conditions of incorrectness for its separate blocks into the sections of the mathematical algorithm using set theory symbols.

4. The use of the basic concepts of set theory during the development of a problem algorithm allows us to formalize the structural and verbal description of the incorrectness of its separate elements, which provides the possibility to form a formal solution to the problem.

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