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Passage of Electromagnetic Waves of Elliptical Polarization through a Flat Dielectric Plate

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Abstract. The article analyzes the passage of electromagnetic waves of elliptical polarization through a flat dielectric plate based on the elliptical wave represented as a sum of horizontal and vertical polarization waves with a quadrature phase shift. In this case, not only amplitude but also phase characteristics of the waves transmitted through the plate are investigated. The results obtained allow one to study the influence of dielectric plates on the amplitudes and phases of electromagnetic waves transmitted through them at random values of the thicknesses and permittivity.

The formulae are obtained for calculating the attenuation and phase of electromagnetic waves of linear and elliptical polarization transmitted through the plate. The change in ellipticity coefficient is examined for elliptical polarization.

The results of the calculation for the 1.6 GHz frequency of the plate of 5 mm in thickness and several values of permittivity are presented.

The calculation is compared with the experiment, and it showed the correctness of the calculations.

The influence of the blisters located in the near zone on the characteristics of a microstrip antenna of the L-band is experimentally studied. It is shown that such blisters practically do not affect the diagrams of an antenna.

Keywords: electromagnetic waves, dielectric plate, blister, attenuation, phase, ellipticity coefficient

Introduction

In today's application of satellite navigation systems, antennas are widely used to receive electromagnetic waves of elliptical polarization. As a rule, these antennas are equipped with radiotransparent protective caps – blisters. Usually, consumer antenna blisters are designed such way that their parts that influence the reception of electromagnetic waves are in the form of flat dielectric plates. When blisters are located as close to the radiators as possible, they enter the near wave zone. In this case, antenna radiators are adjusted to the blisters, and it makes no sense to consider the passage of electromagnetic waves through blisters. Instead, one needs to analyze an antenna with a blister taken into consideration.

The influence of blisters located in the far zone of antennas on their characteristics is studied in sufficient detail in monographs [1–5]. However, most publications refer to antennas and linear polarization waves. The effect of blisters on the properties of rotating polarization antennas has not been studied yet in the literature.

Near receiving antennas, especially on aircraft, there may be structural elements that may affect antenna performance. The antennas are particularly heavily influenced by metal elements. Therefore, it is common to formulate a requirement that antennas should not be shadowed at a certain degree to the horizon. However, dielectric elements can also change antenna characteristics. Hence, it is interesting to investigate passage of electromagnetic waves through dielectric plates when they are located in the far zone of antennas. Passage of linear polarization waves through the dielectric plate is studied in the papers [6–8], but the phases of the passed waves are not determined.

This paper analyzes the diffraction of electromagnetic waves of elliptical polarization on a flat dielectric plate based on the representation of an elliptical wave as a sum of horizontal and vertical phase-shift quadrature polarizations. At the same time not only amplitude, but also phase characteristics of waves passed through the plate are examined. The results obtained allow studying the influence of dielectric plates on amplitudes and phases of the electromagnetic waves which passed through them at any values of thickness and dielectric permeability.

Horizontal polarization

The structure under study is shown in Fig. 1.

We introduce the vector magnetic potential **A** (hereinafter the vector values are indicated in bold). Components of magnetic and electric field are determined from the ratios [7]:

$$\boldsymbol{H} = \frac{1}{\mu} rot\, \boldsymbol{A} \tag{1}$$

$$\boldsymbol{E} = \frac{1}{j\omega\varepsilon} rot \boldsymbol{H} \tag{2}$$

Vector potentials in the regions are represented as incident and reflected waves:

$$\boldsymbol{A_1} = \boldsymbol{A_{1\text{п}}} e^{-j(k_{1x}x+k_{1z}z)} + \boldsymbol{A_{1\text{o}}} e^{j(k_{1x}x+k_{1z}z)} \tag{3}$$

$$\boldsymbol{A_2} = \boldsymbol{A_{2\text{п}}} e^{-j(k_{1x}x+k_{1z}z)} + \boldsymbol{A_{2\text{o}}} e^{j(k_{1x}x+k_{1z}z)} \tag{4}$$

$$\boldsymbol{A_3} = \boldsymbol{A_{3\text{п}}} e^{-j(k_{1x}x+k_{1z}z)} \tag{5}$$

where the lower indexes correspond to the area numbers in Fig. 1.

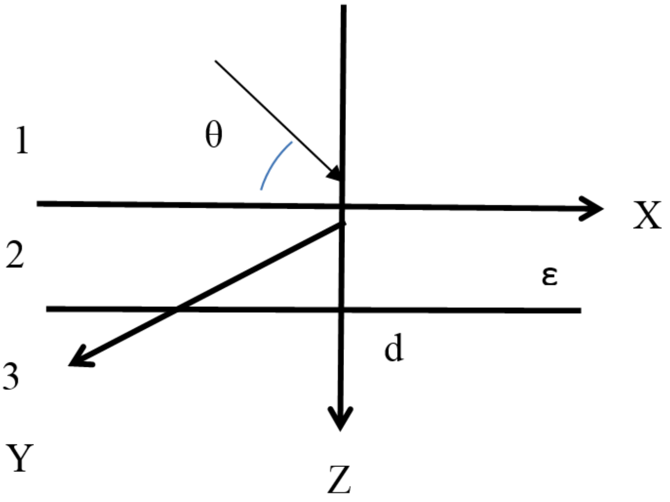

Fig. 1. Fall of electromagnetic wave at the angle θ onto a dielectric plate with the thickness *d* with the dielectric constant ε. Medium *1* and *3* is air, medium *2* is dielectric.

Using formulae (1) and (2), the following expressions are obtained for the magnetic and electric field components necessary for further actions:

$$H_{x1} = j\frac{k_{1z}}{\mu_0}(A_{1\text{п}}e^{-jk_{1z}z} - A_{1\text{o}}e^{jk_{1z}z})e^{-ik_{1x}x} \tag{6}$$

$$H_{x2} = j\frac{k_{2z}}{\mu_0}(A_{1\text{п}}e^{-jk_{2z}z} - A_{1\text{o}}e^{jk_{2z}z})e^{-ik_{1x}x} \tag{7}$$

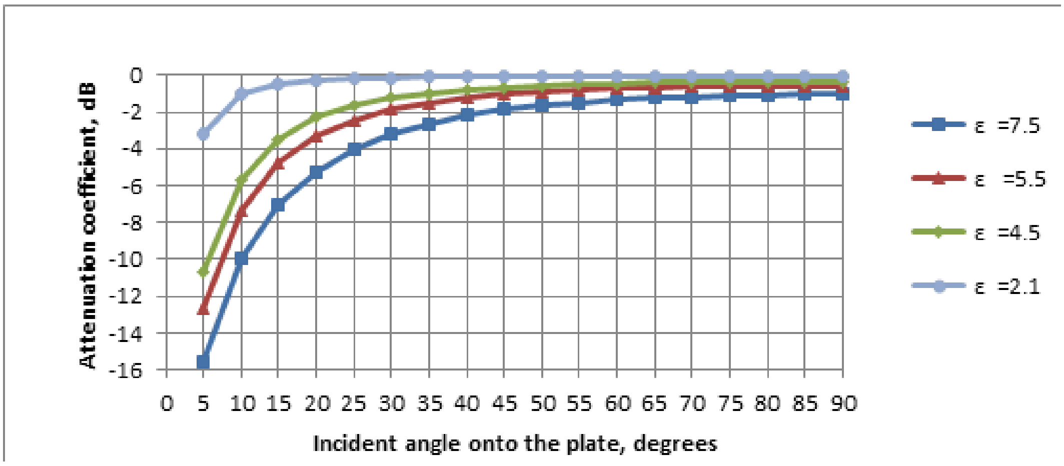

Fig. 2. Attenuation coefficient of electromagnetic wave of linear horizontal polarization, λ=188 mm, by a dielectric plate with the thickness of *d*=5 mm.

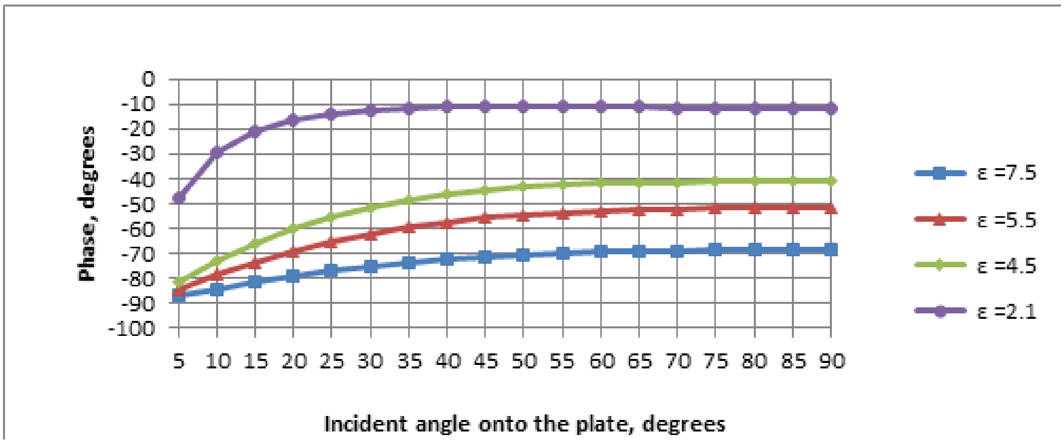

Fig. 3. The phase of linear horizontal polarization electromagnetic wave, λ=188 mm, passed through a dielectric plate with the thickness of *d*=5 mm.

$$H_{x3} = j\frac{k_{1z}}{\mu_0}A_{3п}e^{-jk_{1z}z}e^{-ik_{1x}x} \quad (8)$$

$$E_{y1} = \frac{k_1^2}{j\omega\varepsilon_0\mu_0}(A_{1п}e^{-jk_{1z}z} + A_{1о}e^{jk_{1z}z})e^{-ik_{1x}x} \quad (9)$$

$$E_{y2} = \frac{k_2^2}{j\omega\varepsilon\varepsilon_0\mu_0}(A_{2п}e^{-jk_{2z}z} + A_{2о}e^{jk_{2z}z})e^{-ik_{1x}x} \quad (10)$$

$$E_{y3} = \frac{k_1^2}{j\omega\varepsilon_0\mu_0}A_{3п}e^{-jk_{1z}z}e^{-ik_{1x}x} \quad (11)$$

where ε_0 and μ_0 are electric and magnetic permittivity of vacuum, $k_1^2 = \omega\varepsilon_0\mu_0$ and $k_2^2=\omega\varepsilon\varepsilon_0\mu_0$ are the squares of wave numbers of the corresponding regions, k_{1x}, k_{1z}, k_{2x}, k_{2z} are the constants propagation along the *x* and *z* axes of the corresponding regions.

We will require fulfillment of boundary conditions at the boundaries of the areas representing equality of tangential components of electric and magnetic fields: at $z = 0$, $E_{y1} = E_{y2}$, $H_{x1} = H_{x2}$, at $z = d$, $E_{y2} = E_{y3}$, $H_{x2} = H_{x3}$.

We will obtain a system of algebraic equations:

$$A_{1n} + A_{1o} = A_{2n} + A_{2o} \quad (12)$$

$$k_{1z}(A_{1n} - A_{1o}) = k_{2z}(A_{2n} - A_{2o}) \quad (13)$$

$$A_{2n}e^{-jk2zd} + A_{2o}e^{jk2zd} = A_{3n}e^{-jk1zd} \quad (14)$$

$$k_{2z}(A_{2n}e^{-jk2zd} - A_{2o}e^{jk2zd}) = k_{1z}A_{3n}e^{-jk1zd} \quad (15)$$

Solving this system of equations, we will find the coefficient of passage from the medium *1* into the medium *3*, K_g, and corresponding phase incursion ϕ_g:

$$K_g = \frac{A_{3п}}{A_{1п}} = \frac{e^{jk_{1z}d}}{\cos k_{2z}d + 0.5j(\frac{k_{1z}}{k_{2z}} + \frac{k_{2z}}{k_{1z}})\sin k_{2z}d} \quad (16)$$

$$\phi_g = k_{1z}d - \tan^{-1}[0.5\,(\frac{k_{1z}}{k_{2z}} + \frac{k_{2z}}{k_{1z}})\tan k_{2z}d] \quad (17)$$

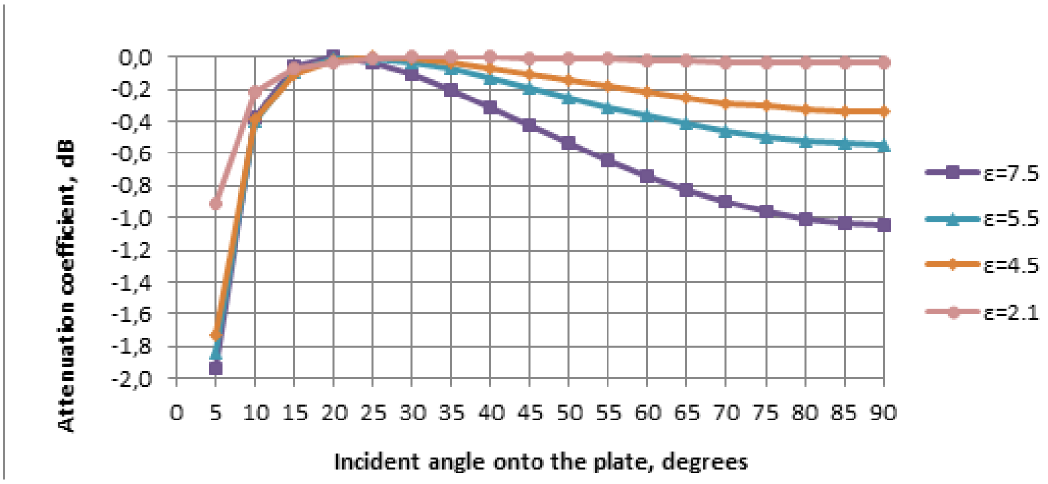

Fig. 4. Attenuation coefficient of electromagnetic wave of linear vertical polarization, λ=188 mm, by a dielectric plate with the thickness of *d*=5 mm.

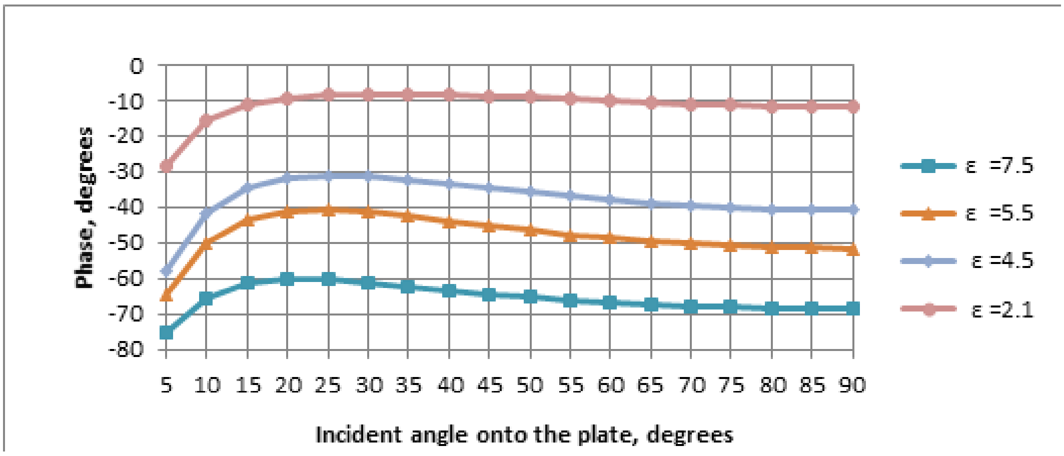

Fig. 5. The phase of electromagnetic wave of linear vertical polarization, λ=188 mm, passed through a dielectric plate with the thickness of *d*=5 mm.

where, according to [4]:

$$k_{2z}d = \frac{2\pi d}{\lambda}\sqrt{\varepsilon - cos^2\theta} \tag{18}$$

$$\frac{k_{2z}}{k_{1z}} = \frac{\sqrt{\varepsilon - cos^2\theta}}{\sin\theta} \tag{19}$$

Figs. 2 and 3 show the results of calculation of attenuation factor and phase according to the formulae (16) and (17) for the wavelength λ=188 mm, *d*=5 mm and several values of dielectric permittivity.

From these figures it can be seen that the attenuation in the cases considered can be as high as 16 dB and the effect on the phase properties of the antenna can also be significant. In particular, changes in the phase of the signal arriving at the antenna in the upper hemisphere can reach –90°.

Vertical polarization

For this case, it is more convenient to introduce a vector electric potential ***F*** through which the components of the electric and magnetic field are defined as follows [7]:

$$\boldsymbol{E} = \frac{1}{\varepsilon} rot\ \boldsymbol{F} \tag{20}$$

$$\boldsymbol{H} = \frac{1}{j\omega\mu} rot\boldsymbol{E} \tag{21}$$

Saving behind electric vector potentials in the areas *1–3* the expressions (3)–(5) and having done conversions similar to the previous section, we will receive the following system of the algebraic equations concerning amplitudes of fields:

$$A_{1n} + A_{1o} = \varepsilon \, (A_{2n} + A_{2o}) \tag{22}$$

$$k_{1z} \, (A_{1n} - A_{1o}) = k_{2z} \, (A_{2n} - A_{2o}) \tag{23}$$

$$\varepsilon \, (A_{2n} e^{-jk2zd} + A_{2o} e^{jk2zd}) = A_{3n} e^{-jk1zd} \tag{24}$$

$$k_{2z} \, (A_{2n} e^{-jk2zd} - A_{2o} e^{jk2zd}) = k_{1z} A_{3n} e^{-jk1zd} \tag{25}$$

Hence, we get the expressions for the transmission factor and the phase incursion:

$$K_v = \frac{A_{3\text{п}}}{A_{1\text{п}}} = \frac{e^{jk_{1z}d}}{\cos k_{2z}d + 0.5j(\varepsilon\frac{k_{1z}}{k_{2z}} + \frac{k_{2z}}{\varepsilon k_{1z}}) \sin k_{2z}d} \tag{26}$$

$$\phi_v = k_{1z}d - \tan^{-1}[\,0.5\,(\varepsilon\frac{k_{1z}}{k_{2z}} + \frac{k_{2z}}{\varepsilon k_{1z}}) \tan k_{2z}d] \tag{27}$$

The ratios (18) and (19) remain true.

Similar to the previous section, Figs. 4 and 5 show the results of calculation of attenuation factor and phase by the formulae (26) and (27) for the wavelength λ=188 mm, *d*=5 mm and several values of dielectric permittivity.

The general nature of the dependences of these values on the incidence angle remains the same as for horizontal polarization. Variation of attenuation in the upper hemisphere of the antenna does not exceed 2 dB, and of the phase –75°.

Elliptical polarization

Let us present the elliptically polarized incident wave on the plate as a sum of quadrature phase-shifted horizontal and vertical components:

$$A_1 = A_g + jA_v = A_g \, (1 + jR_1) \tag{28}$$

where $R_1 = A_v / A_g$ is the coefficient of ellipticity of incident wave. Then the wave which passed through a plate will be defined by the amplitude:

$$A_3 = K_g A_g + jK_v A_v = A_g \, (K_g + jK_v R_1) \tag{29}$$

The passage coefficient of the elliptically polarized wave will be determined as:

$$K_e = \frac{K_g + jK_v R_1}{1 + jR_1} \tag{30}$$

The ellipticity factor of the passed wave will be equal to:

$$R_3 = \frac{K_v}{K_g} R1 \tag{31}$$

and its phase is defined by the expression:

$$\phi_e = k_{1z}d - \tan^{-1}((R_1 \tan\alpha + b) / (a\,R_1 - \tan\alpha)) - \\ - \tan^{-1}(((a+b)\tan\alpha) / (1 - ab\tan^2 k_{2z}d)) - \tan^{-1} R1 \tag{32}$$

where

$$a = 0.5\left(\frac{k_{1z}}{k_{2z}} + \frac{k_{2z}}{k_{1z}}\right) \tag{33}$$

$$b = 0.5(\varepsilon\frac{k_{1z}}{k_{2z}} + \frac{k_{2z}}{\varepsilon k_{1z}}) \tag{34}$$

$$\alpha = k_{2z}d \tag{35}$$

Figs. 6–8 show the results of calculation of attenuation coefficient, ellipticity coefficient and phase by the formulae (30)–(32) for the wavelength λ=188 mm, d=5 mm, and several values of dielectric permittivity.

From these figures it can be seen that the attenuation of the elliptically polarized wave in the upper hemisphere is the mean between horizontal and vertical polarizations. Phase change does not exceed –60°. A dielectric plate has a much stronger influence on the ellipticity of the passed wave: in the cases considered, the ellipticity can decrease from 1 to 0.2.

Attenuation of electromagnetic wave at the frequency of 1.6 GHz by a dielectric plate 5 mm thick with dielectric permittivity 4.5 was experimentally checked. When the plate is located in the far zone of the antenna (at the distance of 10 cm from it), the attenuation of ~0.5 dB is obtained, which corresponds well enough to the calculation (0.35 dB).

Dielectric plate in near antenna field

The effect of the blister located in the near field of the antenna on its characteristics was studied on the electrodynamic model shown in Fig. 9.

The antenna consists of a base with the diameter of 120 mm, substrate from material with the dielectric constant 9.6 and height of 6 mm, diameter of 42 mm, radiator with the diameter of 30 mm of the blister with the thickness of 5 mm located at the height of 3 mm above a radiator (side walls of the blister are not shown). The excited state of the radiator is single-point.

The radiation pattern calculations of this model were performed by the decomposition method followed by the moment method.

At the change ε of the blister, the radiator dimensions changed so that the resonance was observed near the frequency of 1.6 GHz. The results obtained are shown in Table 1.

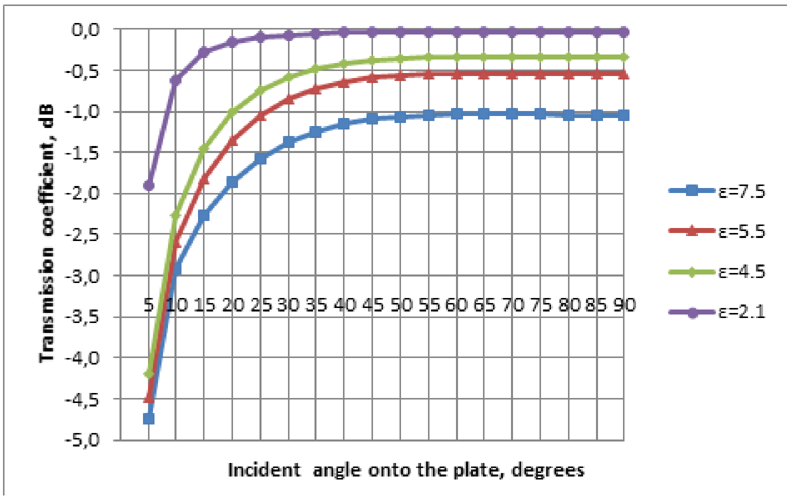

Fig. 6. Attenuation coefficient of electromagnetic wave of elliptical polarization by a dielectric plate with the thickness of *d*=5 mm, λ=188 mm, R_1=1.

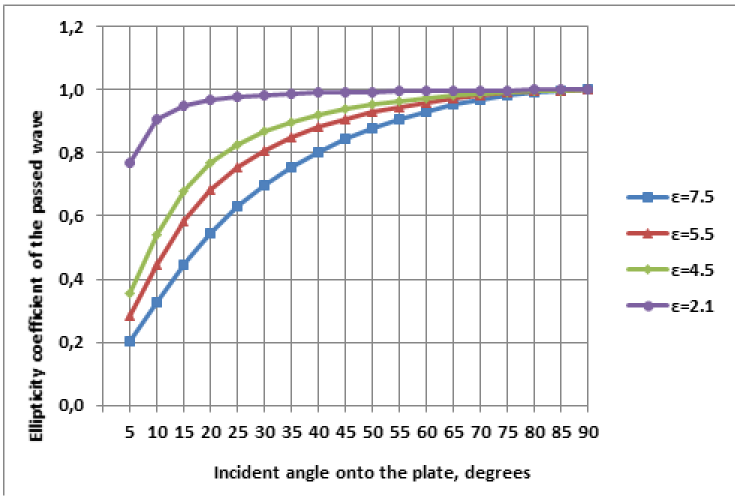

Fig. 7. Ellipticity coefficient of electromagnetic wave of elliptical polarization passed through a dielectric plate with the thickness of *d*=5 mm, λ=188 mm, R_1=1.

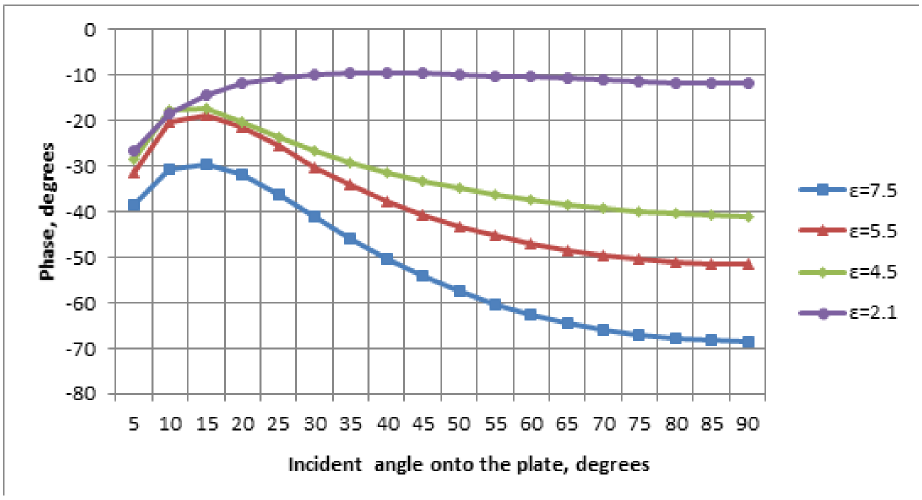

Fig. 8. The phase of electromagnetic wave of elliptical polarization passed through a dielectric plate with the thickness of *d*=5 mm, λ=188 mm, R_1=1.

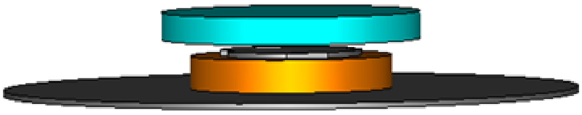

Fig. 9. Electrodynamic model: frequency band microstrip antenna L1.

Table 1. The results of calculation of a microstrip antenna with blisters made of materials with different ε.

ε of the blister	Antenna gain, dB	Ellipticity coefficient at θ = ±90°	Phase change ° at the change of the angle Θ from −90° to 90°
Without blister	6.7	5	8.3
2.1	6.6	4.4	5.7
4.5	6.5	4.5	5.9
5.5	6.3	5	5.8
7.5	6.2	6.4	10.6

As can be seen from the table materials, a blister located in the near field of the antenna practically does not affect its characteristics.

Conclusions

Formulae for calculation of attenuation and phase of electromagnetic waves of linear and elliptical polarizations passed through dielectric plate are obtained. For elliptical polarization, the change in ellipticity coefficient as the plate passes has also been investigated.

Results of calculation for the frequency of 1.6 GHz, a plate 5 mm thick and several values of dielectric permittivity are presented.

A comparison of the calculation with the experiment was made, which showed the correctness of the calculations.

The influence of blisters located in the near zone on the characteristics of the L-band microstrip antenna was experimentally investigated. It is shown that such blisters practically do not affect the diagram properties of the antenna.

References

1. *Obtekateli antenn* [Antenna blisters]. Ed. Shpuntov A.I. Moscow, Izdatel'stvo "Sovetskoye radio", 1950. (in Russian) (translated from English)
2. Kaplun V.A. *Obtekateli antenn SVCH* [Blisters of the microwave antennas]. Moscow, Izdatel'stvo "Sovetskoye radio", 1974, 240 p. (in Russian)
3. Nguayya M.L. *Raschetnyye metody kontrolya sloistykh materialov i izdeliy na osnove issledovaniya koeffitsiyentov otrazheniya i prokhozhdeniya elektromagnitnykh voln: 05.11.13. Dis.... kand. tekhnich. Nauk* [Calculated methods of control of multilayer materials and items based on analysis of reflection and passage coefficients of electromagnetic waves: 05.11.13. (Candidate of Engineering Sciences thesis]. Kazan', KGEU, 2011, 122 p. (in Russian)
4. Minokin M.N. Difraktsiya ploskoy elektromagnitnoy volny na polom dielektricheskom konuse [Diffraction of a flat electromagnetic wave on a hollow dielectric cone]. *Zhurnal radioelektroniki: Elektronnyy zhurnal* [Radio Electronics Journal: Electronic Journal], 2000, No. 6. Available at: http://jre.cplire.ru/jre/jun00/index.html (accessed: May 25, 2019).
5. Kabalin S.V. Issledovaniye kharaktera zavisimosti koeffitsiyenta prokhozhdeniya elektromagnitnoy energii cherez radioprozrachnuyu kryshku ot chastoty radiosignala i tipa polyarizatsii [Study of transmission factor of radio-transparent cap depending on radio wave frequency and type of polarization]. *Antenny* [Antennas]. 2013, No. 1, pp. 41–45. (in Russian)
6. Antonets L.V., Kotov L.N., Shavrov V.G., Shcheglov V.I. Raschet koeffitsiyenta otrazheniya i pogloshcheniya pri naklonnom padenii elektromagnitnykh voln na plastiny [Reflection, transmission, and absorption coefficients calculated for the oblique influence of an electromagnetic wave on a plate]. *Radiotekhnika i elektronika* [Journal of Communications Technology and Electronics], 2008, Vol. 53, No. 4, pp. 389–402. (in Russian)
7. Kugushev A.M., Golubeva N.S. *Osnovy radioelektroniki* [Electronics fundamentals]. Moscow, Izdatel'stvo Energiya, 1969. (in Russian)
8. Gorelov V.V. *Sposob umen'sheniya chastotnoy zavisimosti oslableniya pri prokhozhdenii elektromagnitnoy volny cherez dielektricheskuyuobolochku odnovremenno dlya parallel'noy i perpendikulyarnoy polyarizatsiy i ustroystvo dlya ego realizatsii (vektornyy antennyy obtekatel')* [Method of reducing frequency dependence of attenuation when electromagnetic wave passes through dielectric shell simultaneously for parallel and perpendicular polarizations and device for its implementation (vector antenna blister)]. Patent of the Russian Federation No. 2334315 dated 20.09.2008. (in Russian)