ROCKET-SPACE DEVICE ENGINEERING AND INFORMATION SYSTEMS
Vol. 5, Iss. 2, 2018, pp. 2–9

Estimation of Root-Mean-Square Errors in Measurements of Radio Navigation Parameters

A.V. Molokanov, *contact@spacecorp.ru*
Joint Stock Company "Russian Space Systems", Moscow, Russian Federation
V.E. Vovasov, *Cand. Sci. (Engineering), contact@spacecorp.ru*
Joint Stock Company "Russian Space Systems", Moscow, Russian Federation

Abstract. It is often necessary to evaluate the performance of the user navigation equipment or its compliance with the requirements of the technical task by its measurements. One of the indicators of the quality of the user navigation equipment is the root-mean-square error of the radio navigation parameters.
The article presents a methodology for estimating root-mean-square errors in the measurements of radio navigation parameters obtained by the user navigation equipment installed on spacecraft. The issues of a representative sampling of the number of measurements for determining the root-mean-square errors of measurements eliminating the mutual dynamics of the navigation satellite and user navigation equipment, the dynamics of the receiver time scale shift, as well as the ionospheric component of the measurements are considered. The method includes an estimate of the instability of the receiver reference oscillator.
Based on the technique, the experimental estimates of the root-mean-square errors in pseudorange measurements by code and the carrier phase are made. Application of the developed technique allows evaluating the quality of the user navigation equipment and the correspondence of its characteristics to the specified criteria.

Keywords: GLONASS, GPS, radio navigation parameter, root-mean-square error

Introduction

Rather often, based on the measurements of the user navigation equipment (UNE), it is necessary to estimate the quality of its operation or meeting the requirements of Statement of Work (SOW). One of the indicators of the UNE performance quality is the root-mean-square error (RMS) of radio navigation parameters (RNP). RNP is understood as pseudoranges by the code and pseudoranges by the phase of carrier frequencies. If necessary, it is possible to determine an RMS error and other navigation parameters using the approach described in this article.

Defining the sampling value of measurements

To receive an RMS error, it is necessary to plan an experiment correctly. Firstly, the received measurements are divided into groups. In compliance with [6], the minimum quantity of measurements to define an RMS error, in each group should have not less than 30. In this case the received an RMS error value (x_i) can be considered distributed under the normal law. The number of sessions for receiving the statistics of an RMS error with the set number of reading has to be not less n=10-20 .

At first, we should find an estimate $\widetilde{m}$ for the expectation value of an RMS error:

$$\widetilde{m} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i \quad (1)$$

An RMS error dispersion will be equal to:

$$\widetilde{D} = \frac{\sum_{i=1}^{n} (x_i - \widetilde{m})^2}{n-1} \quad (2)$$

A root-mean-square deviation (RMSD) of an RMS error is defined as:

$$\sigma_{\widetilde{m}} = \sqrt{\frac{\widetilde{D}}{n}} \quad (3)$$

Hence, a confidential interval is as follows:

$$I_\beta = (\widetilde{m} - t_\beta \cdot \sigma_{\widetilde{m}}\, ; \widetilde{m} + t_\beta \cdot \sigma_{\widetilde{m}}) \quad (4)$$

where the value t_β is defined for the normal law as a number of RMSDs which needs to be postponed from the center of dispersion to the right and to the left in order that the probability of getting to the received site was equal to β .

The value t_β is defined by the formula:

$$t_\beta = \sqrt{2} \cdot \Phi^{-1}(\beta) \quad (5)$$

where $\Phi^{-1}(\beta)$ is the function inverse to the Laplace's function. Usually, to ease calculations to obtain t_β, a special Table (14.3.1) [6] is employed. For instance, at the set value $\beta = 0.997$, the value $t_\beta = 3$.

Method to determine an RMS error

The problem of defining the dispersion of an RMS error of the specified parameters is that their mathematical expectation $\widetilde{m}$ is not a constant value in the course of measurements, so application of the traditional expression for dispersion (2) is almost impossible. In this regard, it is offered to make differentiation of the values of the parameters until $\widetilde{m}$ of the received derivatives becomes either a constant or equal to zero. The received RMS error of this derivative is easily recalculated into the RMS error of the initial parameter, since differentiation is a linear operation.

We will bring for an example, the known mathematical expressions for pseudoranges by a high precision (HP) code for the navigation receiver installed on the satellite with the altitude of an orbit of 1000 km in the L1 band for GLONASS [1–3].

$$D_{j,HP}^{L1}(t_i) = R_j(t_i) + c \cdot \Delta T - c \cdot (\Delta T^j) + c\left(T_{ion,L1}^j + \tau_{j,L1,H}\right) - \xi_{L1,H}^j \quad , j = \overline{1, J} \quad (6)$$

by the HP code in the L2 band for GLONASS

$$D_{j,HP}^{L2}(t_i) = R_j(t_i) + c \cdot \Delta T - c \cdot (\Delta T^j + \Delta\tau_n^j) + c\left(T_{ion,L2}^j + \tau_{j,L2,H}\right) - \xi_{L2,H}^j \quad , j = \overline{1, J} \quad (7)$$

where

J is the number of visible GLONASS satellites;

t_i is the moment of measurement formation;

R_j is the path of signal propagation from the phase center of the antenna of the j-th satellite to the phase center of the receiver's antenna equal to

$$R_j(t_i) = \sqrt{\left(x^j - x(t_i)\right)^2 + \left(y^j - y(t_i)\right)^2 + \left(z^j - z(t_i)\right)^2} \quad (8)$$

This is the distance between the points which were occupied by the j-th satellite at the moment of precedence and the receiver at the time of measurement formation.

A measurement formation is understood as a time point, which precedes the moment of measurement formation for the period of signal propagation;

x^j, y^j, z^j are the coordinates of the j-th satellite at the moment of precedence recalculated in that position of the Greenwich system of coordinates which it occupies at the time of measurement of pseudorange;

$x(t_i)$, $y(t_i)$, $z(t_i)$ are the receiver's coordinates at the time of measurement formation;

$T^j_{ion,L1}$, $T^j_{ion,L2}$ is the delay of a code signal of the L1 and L2 bands of the j-th satellite in the ionosphere;

ΔT is the shift of a time scale of the receiver relative to the system time scale of GLONASS;

ΔT^j is the time scale shift of the j-th GLONASS satellite which coincides with the time scale of the L1BT signal relative to the system time scale of GLONASS;

$\tau_{j,L1,HP}$, $\tau_{j,L2,HP}$ is the delay of the code HP signal of the L1 and L2 bands of the j-th GLONASS satellite in the radio frequency part of the receiver;

$\Delta\tau^j_n$ is the shift of the time scale of the L2BT signal relative to the L1BT signal.

$\xi^j_{L1,HP}$, $\xi^j_{L2,HP}$ is the noise component of measurement of pseudoranges by the receiver based on the signal of the L1 and L2 ranges of the HP code of the j-th GLONASS satellite.

Considering the known formulae [1]

$$T^j_{ion,L2} = \gamma \cdot T^j_{ion,L1} \qquad (9)$$

where

$$\gamma = \left(\frac{f^j_{L1}}{f^j_{L2}}\right)^2 \qquad (10)$$

f^j_{L1} is the frequency of the carrier of the signal of the j-th satellite in the L1 band;

f^j_{L2} is the frequency of the carrier of the signal of the j-th satellite in the L2 band.

The value $\gamma = \left(\frac{9}{7}\right)^2$ is for GLONASS.

Considering [5]

$$c \cdot T^j_{ion,L1} = I^j_g \cdot \frac{f^2}{f^2_{j,L1}} \cdot \frac{\alpha}{\sqrt{1 - \left[\frac{R_3}{R_3 + h}\cos\{\eta_j(t_i)\}\right]^2}} \qquad (11)$$

we will obtain the pseudorange shift of the j-th signal in the L1 band caused by the ionosphere.

Here I^j_g is the ionosphere vertical delay of the GLONASS signal at the frequency L1;

R_3 is the radius of the Earth;

$h = 432.5 \cdot 10^3$ m is the altitude of the ionosphere layer where an integral concentration of electrons in the vertical column reaches 50%;

f is the carrier frequency where the estimation I^j_g is received (in the paper it is L1);

$\eta_j(t_i)$ is the elevation of the j-th navigation satellite relative to the receiver;

α is the coefficient taking into account the decrease in the sum concentration of electrons in the ionosphere column due to the object is located not on the Earth surface.

Allowing for (11), we will rewrite (6) and (7) as following:

$$D^{L1}_{j,BT}(t_i) = R_j(t_i) + c \cdot \Delta T - c \cdot (\Delta T^j) + I^j_g \cdot \frac{\alpha}{\sqrt{1 - \left[\frac{R_3}{R_3 + h}\cos\{\eta_j(t_i)\}\right]^2}} + c \cdot \tau_{j,L1,BT} - \xi^j_{L1,BT} \qquad (12)$$

$$D^{L2}_{j,BT}(t_i) = R_j(t_i) + c \cdot \Delta T - c \cdot (\Delta T^j + \Delta\tau^j_n) + I^j_g \cdot \gamma \cdot \frac{\alpha}{\sqrt{1 - \left[\frac{R_3}{R_3 + h}\cos\{\eta_j(t_i)\}\right]^2}} + c \cdot \tau_{j,L2,BT} - \xi^j_{L2,BT} \qquad (13)$$

It is obvious that on the interval corresponding to the group of changes such parameters as ΔT^j, $\Delta\tau^j_n$, $\tau_{j,L2,BT}$, $\tau_{j,L1,BT}$, $\eta_j(t_i)$, can be considered constants while the values $R_j(t_i)$, ΔT, $\alpha \cdot I^j_g$ have a significant dynamics.

The known mathematical expressions for pseudorange at the phase of the carrier for the navigation receiver installed on the satellite with the orbit altitude 1000 km [1, 3] should be given.

$$G^{L1}_j(t_i) = -c \cdot (\Delta T^j + \Delta\tau^j_{L1}) + c \cdot \Delta T + R_j(t_i) + \lambda^j_{L1}\left(\varphi_{0,L1} + \varphi^j_{0,L1} + \varsigma^{L1}_{\psi_j}\right) - c\left(T^j_{ion,L1}\right) - \lambda^j_{L1}\left(\varphi^j_{h,L1}\right) - M^{L1}_j \cdot \lambda^j_{L1} \qquad (14)$$

$$G^{L2}_j(t_i) = -c \cdot (\Delta T^j + \Delta\tau^j_n + \Delta\tau^j_{L2}) + c \cdot \Delta T + R_j(t_i) + \lambda^j_{L2}\left(\varphi_{0,L2} + \varphi^j_{0,L2} + \varsigma^{L2}_{\psi_j}\right) - c\left(T^j_{ion,L2}\right) - \lambda^j_{L2}\left(\varphi^j_{h,L2}\right) - M^{L2}_j \cdot \lambda^j_{L2} \qquad (15)$$

where λ^j_{L1}, λ^j_{L2} is the wavelength of the carrier of the *j*-th satellite in the L1 and L2 bands;

$\varphi_{0,L1}$, $\varphi_{0,L2}$ is the initial phase of the receiver in the L1 and L2 bands;

$\varphi^j_{0,L1}$, $\varphi^j_{0,L2}$ is the indefinite initial radiation phase of the *j*-th satellite in the L1 and L2 bands;

$\Delta\tau^j_{L1}=(\tau^j_{L1,CT} - \tau^j_{L1,BT})$ is the delay of the average precision (AP) code relative to HP in the L1 range in the equipment of the GLONASS satellite;

$\Delta\tau^j_{L2}=(\tau^j_{L2,CT} - \tau^j_{L2,BT})$ is the delay of the AP code relative to HP in the L2 range in the equipment of the GLONASS satellite;

$\varphi^j_{h,L1}$, $\varphi^j_{h,L2}$ is the phase aperture signal distortions of the *j*-th satellite in the receiver in the L1 and L2 bands;

M^{L1}_j, M^{L2}_j is the indefinite integer being nonuniqueness of phase measurements of the signal of the *j*-th satellite in the receiver in the L1 and L2 bands;

$\varsigma^{L1}_{\psi_j}$, $\varsigma^{L2}_{\psi_j}$ is the noise term of the measurement of the signal pseudopahse of the *j*-th satellite in the receiver in the L1 and L2 ranges.

Considering (11), we will rewrite (14) and (15) as following:

$$G^{L1}_j(t_i) = -c\cdot(\Delta T^j + \Delta\tau^j_{L1}) + c\cdot\Delta T + R_j(t_i) + \lambda^j_{L1}\left(\varphi_{0,L1} + \varphi^j_{0,L1} + \varsigma^{L1}_{\psi_j}\right) - I^j_g\cdot\frac{\alpha}{\sqrt{1-\left[\frac{R_3}{R_3+h}\cos\{\eta_j(t_i)\}\right]^2}} - \lambda^j_{L1}\left(\varphi^j_{h,L1}\right) - M^{L1}_j\cdot\lambda^j_{L1} \quad (16)$$

$$G^{L2}_j(t_i) = -c\cdot(\Delta T^j + \Delta\tau^j_n + \Delta\tau^j_{L2}) + c\cdot\Delta T + R_j(t_i) + \lambda^j_{L2}\left(\varphi_{0,L2} + \varphi^j_{0,L2} + \varsigma^{L2}_{\psi_j}\right) - I^j_g\cdot\gamma\cdot\frac{\alpha}{\sqrt{1-\left[\frac{R_3}{R_3+h}\cos\{\eta_j(t_i)\}\right]^2}} - \lambda^j_{L2}\left(\varphi^j_{h,L2}\right) - M^{L2}_j\cdot\lambda^j_{L2} \quad (17)$$

It is obvious that on the interval corresponding to the measurements such parameters as ΔT^j, $\Delta\tau^j_n$, $\eta_j(t_i)$, $\Delta\tau^j_{L1}$, $\lambda^j_{L1}\cdot(\varphi_{0,L1} + \varphi^j_{0,L1})$, $\lambda^j_{L1}\cdot(\varphi^j_{h,L1})$, $M^{L1}_j\cdot\lambda^j_{L1}$ can be considered constants while the values $R_j(t_i)$, ΔT, $\alpha\cdot I^j_g$ have a significant dynamics. As it was already specified, we will make differentiation of the received measurements for decrease in dynamics. Derivatives are needed to be taken until the dynamic error does not turn out to be less than noise. Since measurements of pseudorange based on the carrier phase are the most accurate, they will demand the highest derivative. Elimination of the dynamics of navigation spacecraft and the satellite with the height of an orbit of 1000 km, dynamics of the drift of the time scale of the receiver and also an ionospheric component of the expressions of pseudoranges, as the experiment has shown, requires receiving the sixth derivative of the measured pseudorange on the carrier phase. To sum up the results, we will use the sixth derivative at measurements of pseudorange by the code. In this case the values of the measurements $D^{L1}_{j,HP}(t_i)$, $D^{L2}_{j,HP}(t_i)$, $G^{L1}_j(t_i)$ and $G^{L2}_j(t_i)$ will be written as $G_j(t_i)$. An approximate value of the six derivative can be estimated with the following polynominal:

$$\Delta G_j(t_i) = G_j(t_{i+6}) - 6\cdot G_j(t_{i+5}) + 15\cdot G_j(t_{i+4}) - 20\cdot G_j(t_{i+3}) + 15\cdot G_j(t_{i+2}) - 6\cdot G_j(t_{i+1}) + G_j(t_i) \quad (18)$$

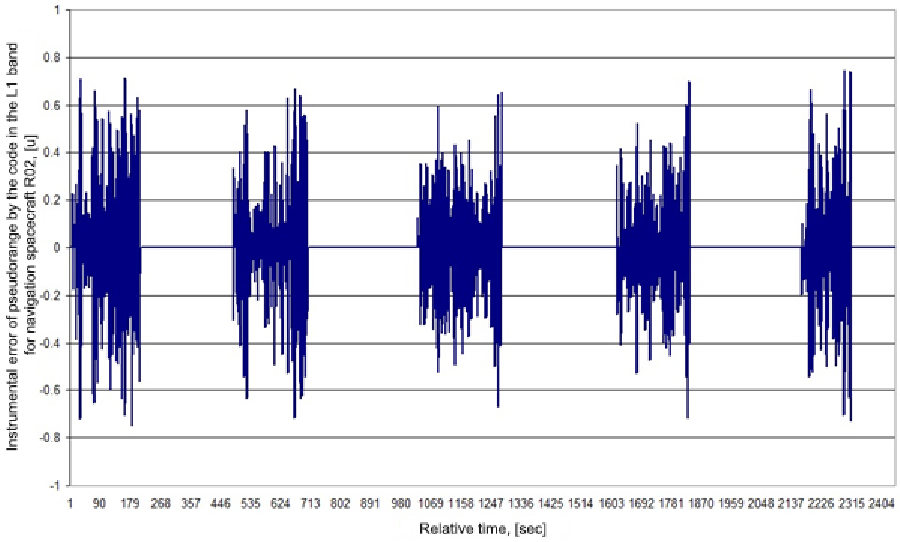
The dispersion of differences $\sigma^2_{\Delta G_j}$ will be equal to

$$\sigma^2_{\Delta G_j} = \frac{1}{N}\sum_{i=1}^{N}[\Delta G_j(t_i)]^2 = 924\cdot\sigma^2_{G_j} \quad (19)$$

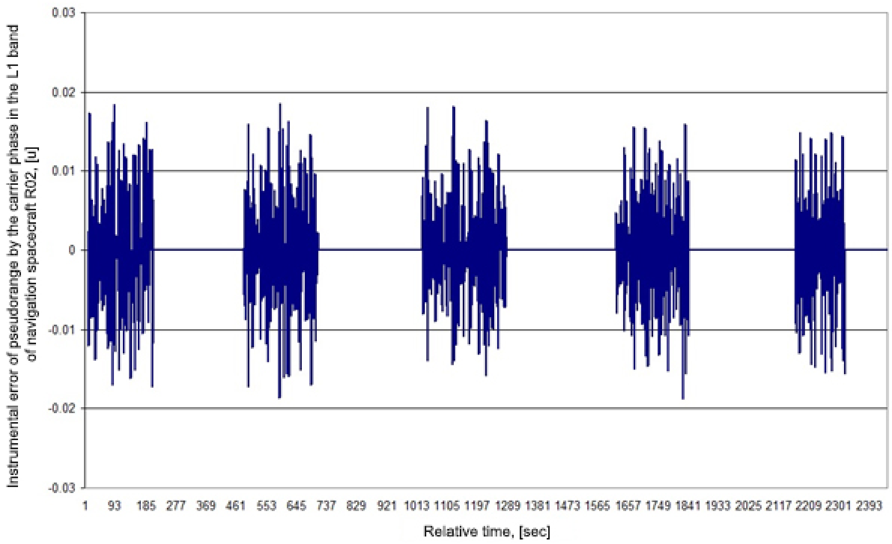
Hence, an RMS error of measurements $G_j(t_i)$ will be equal to

$$\sigma_{G_j} = \sqrt{\sigma^2_{\Delta G_j}/924} \quad (20)$$

As an example of the use of the expression (18), Fig. 1 gives the graph of the change of the calculated σ_{G_j} pseudorange GLONASS of the L1 band by the code (**a**) and phase (b) of the carrier applying the six derivative. The Figure shows the relative time as a quantity of 10 second readings during a navigation session. A dashed graph is stipulated by the conditions of the navigation spacecraft by the antenna system UNE located on the satellite with the orbit about 1000 km.



(a)



(b)

Fig. 1. An instrumental RMS error of measurement of pseudorange GLONASS of the L1 band by the code (a) and by the phase (b) of the carrier frequency.

Method to determine an RMS error regardless the noises of the reference generator

For the UNE developer, it is very important to be convinced of correctness of operation of the receiver. Usually at creation of UNE, a reference generator which noises are much lower than thermal noise of the receiver is chosen. However because of different errors of creation of the receiver it is important to know a contribution of noise of the reference generator to an error of measurements of pseudorange by the phase carrier. Definition of RMS errors of measurements of pseudoranges by the phase carrier regardless of the noises of the master oscillator is used for this purpose.

As it is necessary to determine RMS errors of measurements of pseudoranges by the carrier phase without the noises of the master oscillator, one should receive the expression where there is no value ΔT . To do this, using the difference of the expressions (16) and (17), we will receive

$$\Delta G_j(t_i) = G_j^{L2}(t_i) - G_j^{L1}(t_i) =$$
$$= -c\cdot(\Delta\tau_n^j + \Delta\tau_{L2}^j - \Delta\tau_{L1}^j) +$$
$$+(\gamma-1)\cdot I_g \cdot \frac{\alpha}{\sqrt{1-\left[\frac{R_3}{R_3+h}\cos\{\eta_j(t_i)\}\right]^2}} +$$
$$+\lambda_{L2}^j\left(\varphi_{0,L2} + \varphi_{0,L2}^j - \varphi_{h,L2}^j + \varsigma_{\psi_j}^{L2}\right) -$$
$$-\lambda_{L1}^j\left(\varphi_{0,L1} + \varphi_{0,L1}^j - \varphi_{h,L1}^j + \varsigma_{\psi_j}^{L1}\right) +$$
$$+ M_j^{L1}\cdot\lambda_{L1}^j - M_j^{L2}\cdot\lambda_{L2}^j \qquad (21)$$

It is experimentally determined that for the navigation receiver installed on the satellite with the altitude of an orbit of 1000 km, the second derivative of the difference of pseudoranges by the carrier phase is almost equal to zero. The approximate value of the second derivative can be received by means of the following polynomial.

$$\Delta\Delta G_j(t_i) = \Delta G_j(t_{i+1}) - 2\cdot\Delta G_j(t_i) + \Delta G_j(t_{i-1})$$

We should determine the pseudorange dispersion by the carrier phase in the L1 band as $\sigma^2_{G_j^{L1}} = [\lambda_{L1}^j]^2 \cdot \sigma^2_{\varsigma_{\psi_j}^{L1}}$ and in the L2 band as $\sigma^2_{G_j^{L2}} = [\lambda_{L2}^j]^2 \cdot \sigma^2_{\varsigma_{\psi_j}^{L2}}$.

The dispersion of the $\sigma^2_{\Delta\Delta G_j(t_i)}$ value will be equal to

$$\sigma^2_{\Delta\Delta G_j(t_i)} = \frac{1}{N}\sum_{i=1}^{N}[\Delta\Delta G_j(t_i)]^2 =$$
$$= \frac{1}{N}\sum_{i=1}^{N}\left[\Delta G_j(t_{i+1}) - 2\cdot\Delta G_j(t_i) + \Delta G_j(t_{i-1})\right]\times$$
$$\times\left[\Delta G_j(t_{i+1}) - 2\cdot\Delta G_j(t_i) + \Delta G_j(t_{i-1})\right] =$$
$$= \frac{1}{N}\sum_{i=1}^{N}[\Delta G_j(t_{i+1})]^2 + \frac{4}{N}\sum_{i=1}^{N}[\Delta G_j(t_i)]^2 +$$
$$+\frac{1}{N}\sum_{i=1}^{N}[\Delta G_j(t_{i-1})]^2 = \frac{6}{N}\sum_{i=1}^{N}[\Delta G_j(t_i)]^2 =$$
$$= 6\cdot\sigma^2_{\Delta G_j(t_i)} \qquad (22)$$

Here a statistical independency of readings of pseudoranges by the carrier phase is considered as well as their similar dispersion during the session.

The dispersion of differences $\Delta G_j(t_i)$ is to be determined:

$$\sigma^2_{\Delta G_j(t_i)} = \frac{1}{N}\sum_{i=1}^{N}[\Delta G_j(t_i)]^2 =$$
$$= \frac{1}{N}\sum_{i=1}^{N}[\Delta G_j(t_i)]^2 = \frac{1}{N}\sum_{i=1}^{N}[G_j^{L2}(t_i) - G_j^{L1}(t_i)]^2 =$$
$$= \frac{1}{N}\sum_{i=1}^{N}[G_j^{L2}(t_i)]^2 + \frac{1}{N}\sum_{i=1}^{N}[G_j^{L1}(t_i)]^2 = \sigma^2_{G_j^{L2}} + \sigma^2_{G_j^{L1}} \qquad (23)$$

Hence,

$$\sigma^2_{\Delta\, G_j(t_i)} = 6\cdot\sigma^2_{G_j^{L2}} + 6\cdot\sigma^2_{G_j^{L1}} \qquad (24)$$

The dispersions of pseudophases in the L_1 and L_2 bands caused by the noises of the equipment, which in case are determined by the expressions [1,4]:

$$\sigma^2_{G_j^{L1}} = [\lambda_{L1}^j]^2 \cdot \left[\frac{\Delta f_{C\,\acute{O}}\left(1+\frac{1}{2\cdot k\cdot q_{c/n_0,L1}\cdot T'}\right)}{k\cdot q_{c/n_0,L1}}\right] \qquad (25)$$

$$\sigma^2_{G_j^{L2}} = [\lambda_{L2}^j]^2 \cdot \left[\frac{\Delta f_{C\,\acute{O}}\left(1+\frac{1}{2\cdot k\cdot q_{c/n_0,L2}\cdot T'}\right)}{k\cdot q_{c/n_0,L2}}\right] \qquad (26)$$

where

$\Delta f_{ССФ}$ = 25 Hz, the noise band of PLL;

k is the reserve coefficient;

$q_{c/n_0,L1}$, $q_{c/n_0,L2}$ is the energy potential of the radio link in the L_1 and L_2 bands;

T' = 1 sec is the time for accumulation of information parameters of the digital receiver.

Taking into account the dependences $q_{c/n_0,L1} = \beta\cdot q_{c/n_0,L2}$ will be written as

$$\sigma^2_{\Delta\Delta G_j(t_i)} \approx 6 \cdot [\lambda^j_{\mathrm{L1}}]^2 \times$$
$$\times \left[\frac{\Delta f_{\mathrm{CC\Phi}} \left(1 + \dfrac{1}{2 \cdot k \cdot q_{c/n_0,\mathrm{L1}} \cdot T'} \right)}{k \cdot q_{c/n_0,\mathrm{L1}}} \right] \cdot (1 + \gamma \cdot \beta) \approx$$
$$\approx 6 \cdot [\lambda^j_{\mathrm{L2}}]^2 \cdot \left[\frac{\Delta f_{\mathrm{CC\Phi}} \left(1 \right) \dfrac{1}{2 \cdot k \cdot q_{c/n_0,\mathrm{L2}} \cdot T'})}{k \cdot q_{c/n_0,\mathrm{L2}}} \right] \times$$
$$\times \left(1 + \frac{1}{\gamma \cdot \beta} \right) \qquad (27)$$

Hence, we receive the values of RMS error pseudoranges by the carrier phase in the L_1 and L_2 bands.

$$\sigma_{G_j^{\mathrm{L1}}} \approx \left[\frac{\sigma^2_{\Delta\Delta G_j(t_i)}}{6 \cdot (1 + \gamma \cdot \beta)} \right]^{0,5} \qquad (28)$$

$$\sigma_{G_j^{\mathrm{L2}}} \approx \left[\frac{\sigma^2_{\Delta\Delta G_j(t_i)}}{6 \cdot (1 + \gamma \cdot \beta)} \cdot \gamma \cdot \beta \right]^{0,5} \qquad (29)$$

As an example of using the expression (28) in the Fig. 2, the graph of the change of the calculated σ_{G_j} pseudorange GLONASS of the L1 band by the carrier phase regardless the noises of the reference oscillator is given. In the Figure, the relative time is a number of 10-second readings during navigation session. A dashed graph is explained by the conditions of the visibility zone of the navigation spacecraft by the antenna system of UNE located on the satellite with the orbit of about 1000 km.

Assessment of instability of the reference generator

In case of obtaining an essential difference in RMS errors of pseudoranges by the carrier phase in the corresponding range taking into account noises of the reference generator and without the noises, it is possible to approximately estimate instability of the reference generator under operation.

The known formula of Doppler frequency

$$F_{\mathrm{D}} = -F_0 \cdot \frac{V}{c}$$

where

F_0 is the carrier frequency, V is the radial velocity, c is the light speed.

Hence, frequency errors are connected with speed errors as follows:

$$\Delta F_{\mathrm{D}} = -F_0 \cdot \frac{\Delta V}{c}$$

Hence,

$$\frac{\Delta F}{F_0} = \frac{|\Delta F_{\mathrm{D}}|}{F_0} = \frac{|\Delta V|}{c}$$

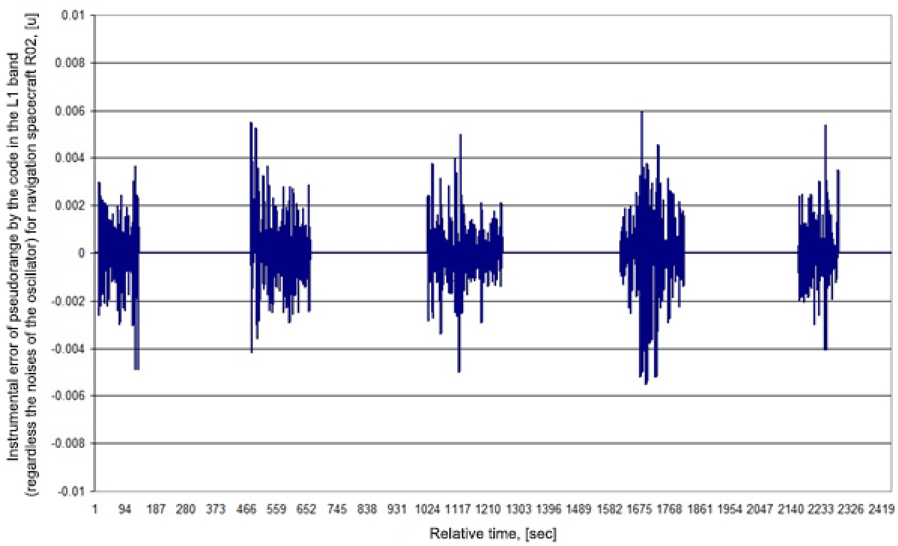


Fig. 2. Instrumental RMS error of measurement of pseudorange GLONASS of the L1 band by the carrier phase (regardless the noises of the reference oscillator)

Here $\frac{\Delta F}{F_0}$ is a short-term instability of the reference generator.

The speed in the moment of time *i* is connected with the range by the following ratio: $V_i = \dot{D}_i = \frac{D_{i+1} - D_{i-1}}{2 \cdot \Delta t}$

Here D_{i+1}, D_{i-1} are the readings of pseudorange by the phase in *i* + 1 and *i* – 1 moments of time,

Δt is the time interval between the readings (this interval is equal to 1 second in the receiver).

In this case, the error of the speed will be determined through the RMS error of pseudorange (σ_D) obtained regardless the noises of the reference generator and the RMS error of pseudorange (σ_{D+}) obtained taking into account the noises of the reference generator in the form:

$$\Delta V = \frac{\sqrt{\sigma_{D+}^2 - \sigma_D^2}}{\sqrt{2} \cdot \Delta t}$$

Hence,

$$\frac{\Delta F}{F_0} = \frac{|\Delta V|}{c} = \frac{\sqrt{\sigma_{D+}^2 - \sigma_D^2}}{\sqrt{2} \cdot c \cdot \Delta t} \quad (30)$$

One should take a right border of the confidence limit as the values of the RMS error of pseudoranges. This approach is valid when $\sigma_{D+}^2 > \sigma_D^2$.

Conclusion

The technique is developed to determine RMS errors of measurements of radio navigation parameters determined by the UNE. In the developed technique the selection of the number of measurements of radio navigation equipment sufficient for evaluation of a RMS error, definition of an RMS error of measurements of radio navigation equipment taking into account and without instability of the reference generator of the receiver is performed, and assessment of instability of the reference generator is given. The assessment of operation quality of the UNE and compliance of its characteristics to the set criteria is the result of application of this technique.

The carried out experimental estimates by the offered technique of definition of RMS errors clearly demonstrate the influence of instability of the reference generator on characteristics of UNE.

References

1. *GLONASS. Printsipy postroeniya i funktsionirovaniya* [GLONASS. Principles of construction and operation]. Moscow, Radiotekhnika, 2010, 800 p. Eds. Perov A.I., Kharisov V.N. 4th ed. rev. (in Russian)
2. Povalyaev A.A. *Sputnikovye radionavigatsionnye sistemy: vremya, pokazaniya chasov, formirovanie izmereniy i opredelenie otnositel'nykh koordinat* [Satellite radio navigation systems: time, clock readings, formation of measurements and determination of relative coordinates]. Moscow, Radiotekhnika, 2008, 328 p. (in Russian)
3. Povalyaev A.A., Veytsel' V.A., Mazepa R.B. *Global'nye sputnikovye sistemy sinkhronizatsii i upravleniya v okolozemnom prostranstve: ucheb. posobie* [Global satellite systems for synchronization and control in near-Earth space: Textbook]. Moscow, Vuzovskaya kniga, 2012, 188 p. Ed. Povalyaev A.A. (in Russian)
4. *Statisticheskaya teoriya radiotekhnicheskikh sistem. Uchebnoe posobie dlya vuzov.* [Statistical theory of radio engineering systems. Textbook for universities]. Moscow, Radiotekhnika, 2003, 400 p. (in Russian)
5. Vovasov V.E., Ipkaev N.B. Metodika opredeleniya apparaturnykh zaderzhek signala dlya dvukhchastotnogo priemnika SRNS GLONASS [Technique of Definition of Hardware Delays of a Signal for Two-Frequency Receiver SRNS GLONASS]. *Raketno-kosmicheskoe priborostroenie i informatsionnye sistemy* [Rocket-Space Device Engineering and Information Systems]. 2014, Vol. 1, No. 2, pp. 25–32. (in Russian)
6. Venttsel' E.S. *Teoriya veroyatnostey* [Probability theory]. Moscow, Nauka, 1968, 576 p. (in Russian)