Digital Phasing to Increase the Efficiency of the B-529 Antenna System

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Abstract: This article describes the proposals for practical achievements of the ideal signal-to-noise ratio in the receive path of the B-529 antenna system by means of digital phasing of four antennas of this system. A digital equisignal forming method of difference guidance signals from the sum signal from four antennas of the B-529 system is proposed. It is stressed that this method allows to increase twice a signal-to-noise ratio in the antenna guidance path in comparison with a traditional guidance method from sum and difference signals. It is reported that the results can be used when modernizing the B-529 antenna system through transferring its receive path and guidance path to digital processing of radio signals.

Keywords: antenna system, hased array, receiver, digital phasing, intermediate frequency, path-length difference, equisignal method, sum and difference signal.

This article gives a comparative analysis of analog and digital methods of phasing of signals from four antennas of the B-529 system, mounted on a common frame and forming a phased array antenna (see Fig. 1).

Fig. 1. B-529 array antenna

The schematic diagram of a phased B-529 array antenna is presented in Fig. 2.

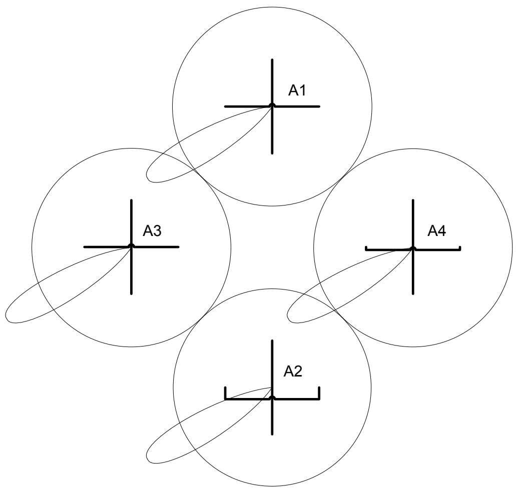

Fig. 2. Schematic diagram of B-529 array antenna

Comparison of the analog and digital methods of processing and phasing of signals from the four antennas of the B-529 system will be conducted under the same conditions, with low-noise amplifiers installed directly on the feed elements of the antennas. Losses in the cables, connected to the hybrid rings from 1 dB in the VHF band up to 2-2.4 dB in the higher UHF band can be neglected.

As seen in Figure 2, the top antenna will be referred to as A1, the bottom one as A2, the left one as A3 and the right one as A4; Ψ is the azimuth, Φ_{SC} is the elevation angle to the direction of the spacecraft, $\Delta\Psi$ is the azimuth deviation, $\Delta\Phi$ is the deviation of the elevation angle from the direction of the SC. During the analysis it is assumed that the array phase center of B-529 is located at the intersection of the lines connecting the phase centers of A1 – A2 and A3 – A4 antennas.

Figure 3 shows the geometric model for calculation of phase incursions in A1 – A4 antennas in relation to the array phase center, occurring when the radiation pattern axis of the antenna declines from the direction of the SC by $\Delta\Psi$ in azimuth and by $\Delta\Phi$ in elevation.

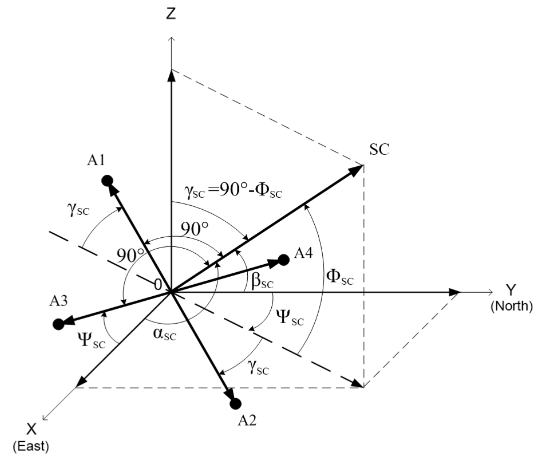

Fig. 3. Geometric model of the B-529 array antenna

The path length difference from the SC to the ΔR_{i0} antenna in relation to the array phase center equals the length of projection of the phase center vector of the antenna on the SC directivity vector and is calculated by the formula for angles between the vectors through directional cosines:

$$\Delta R_{i0} = L_{i0}\cdot[\cos(a_{Ai})\cdot\cos(\alpha_{SC}) + \cos(b_{Ai})\cdot\cos(\beta_{SC}) + \cos(g_{Ai})\cdot\cos(\gamma_{SC})] \quad (1)$$

Here L_{i0} is the distance from the array phase center to the phase center of the antenna Ai. With antenna diameter D = 6 m, $L_{i0} = D/\sqrt{2} = 4.24$ m.

The directional cosine angles in the direction of the SC and the A1 – A4 antennas are determined by the simple formulas:

$$\cos(\alpha_{SC}) = \cos(\Phi_{SC})\cdot\sin(\Psi_{SC});\ \cos(\beta_{SC}) = \cos(\Phi_{SC})\cdot\cos(\Psi_{SC});\ \cos(\gamma_{SC}) = \sin(\Phi_{SC}); \quad (2)$$

$$\cos(a_{A1}) = -\sin(\Phi_{RP})\cdot\sin(\Psi_{RP});\cos(b_{A1}) = -\sin(\Phi_{RP})\cdot\cos(\Psi_{RP});\ \cos(g_{A1}) = \cos(\Phi_{RP}); \quad (3)$$

$\cos(a_{A2}) = \sin(\Phi_{RP})\cdot\sin(\Psi_{RP});\cos(b_{A2}) =$

$\sin(\Phi_{RP})\cdot\cos(\Psi_{RP});\ \cos(g_{A2}) = -\cos(\Phi_{RP});$ (4)

$\cos(a_{A3}) = \cos(\Psi_{RP});\cos(b_{A1}) =$

$= -\sin(\Psi_{RP});\ \cos(g_{A4}) = 0;$ (5)

$\cos(a_{A4}) = -\cos(\Psi_{RP});\cos(b_{A1}) =$

$= \sin(\Psi_{RP});\ \cos(g_{A4}) = 0;$ (6)

$\Phi_{RP} = \Phi_{SC} + \Delta\Phi; \qquad \Psi_{RP} = \Psi_{SC} + \Delta\Psi;$ (7)

The path length difference ΔR_{ij} and propagation time ΔT_{ij} of the beams between the antennas Ai and Aj, i,j = 1 - 4, i ≠ j is given by:

$\Delta R_{ij} = \Delta R_{i0} - \Delta R_{i0}$ (8)

$\Delta T_{ij} = \Delta R_{ij}/c,$ (9)

where *c* is the speed of light.

The phase shift between the signals of the Ai and Aj antennas on the carrier frequency is given by:

$$\Delta\varphi_{ij} = \omega_c \cdot \Delta T_{ij} = 2\cdot\pi\cdot f_c \cdot \Delta T_{ij} \tag{10}$$

The acquisition of the sum and difference signals from the vertical and horizontal antennas of the B-529 array in each of the operating bands M1, M2, D1, D2, D4 is performed on the circuit of hybrid ring couplers [3, pp. 94 – 95], that are essentially a coil of coaxial transmission line cable, 3λ/2 long with four ports each placed one quarter wavelength away from each other, as shown in Figure 4. When the port 2 is exited, waves travel in both directions through the ring, at the ports 3 and 1 the signal is in phase, and at the port 4 it is in anti-phase. Thus, the signal power is split equally between the ports 1 and 3, and the port 4 is isolated [3, p. 95].

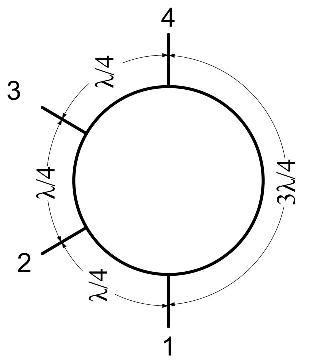

Fig. 4. A hybrid ring on a coaxial cable line

When the port 4 is exited, the waves traveling in both directions through the ring are also in phase at the ports 3 and in anti-phase at the port 2. Therefore, in this case the signal power is split equally between the ports 1 and 3, and the port 2 is isolated. When the ports 2 and 4 are excited simultaneously with an in-phase signal, the waves at the port 3 coming from the ports 2 and 4 are in phase and summarized in amplitude, while the waves at port 1 are in anti-phase and subtracted amplitude.

With the equal power and an amplitude of $\sqrt{2RPs}$ for each of the in-phase signals at the ports 2 and 4, the port 3 receives in-phase signals and the port 1 receives counterphase signals with a power of $P_s/2$ and an amplitude of $\sqrt{RPs}$. Here R is the loading impedance, matched with the iterative impedance of the line. At the port 3 the incoming signals combine and have an amplitude of $\sqrt{\boldsymbol{R}}$ s and a resulting power of $2P_{c,}$ and at the port 1 the signals annihilate and their resulting power is 0. Therefore, the power of the signal at the port 3 equals the sum power of the in-phase signals, fed into the ports 2 and 4. The sum and differential waveform generation in the B-529 antenna array is based on this feature of the hybrid ring.

The noise power P_n of the input ports 2 and 4 is divided equally between the ports 1 and 3, and since the noise signals are independent, they are summed in power at the ports 2 and 4. Therefore, the noise signals with a power of P_n from the ports 2 and 4 give a noise signal of a singular power P_n at the ports 3 and 1.

This results in the doubling of the noise-to-signal ratio when in-phase signals are summed on the hybrid ring: $P_{s3}/P_{n3}=2(P_s/P_n)$.

The schematic diagram of the analogue forming of sum and difference signals of plane-linear polarization individually for the vertical and horizontal polarization from the B-529 antenna array is shown in Figure 5. The diagram features two stages of signal combining with in-phase signals being combined on both stages. Therefore, the signal to noise ratio is doubled on each stage with the resulting signal to noise ratio of $4(P_s/P_n)$.

It is assumed that after forming of the sum and difference signals at the hybrid rings, the further signal processing is performed on modern digital receivers. Since the power of the thermal noise is given by the formula:

$$P_n = k\cdot t^{\circ}\cdot \Delta F_{LNA}\ , \tag{11}$$

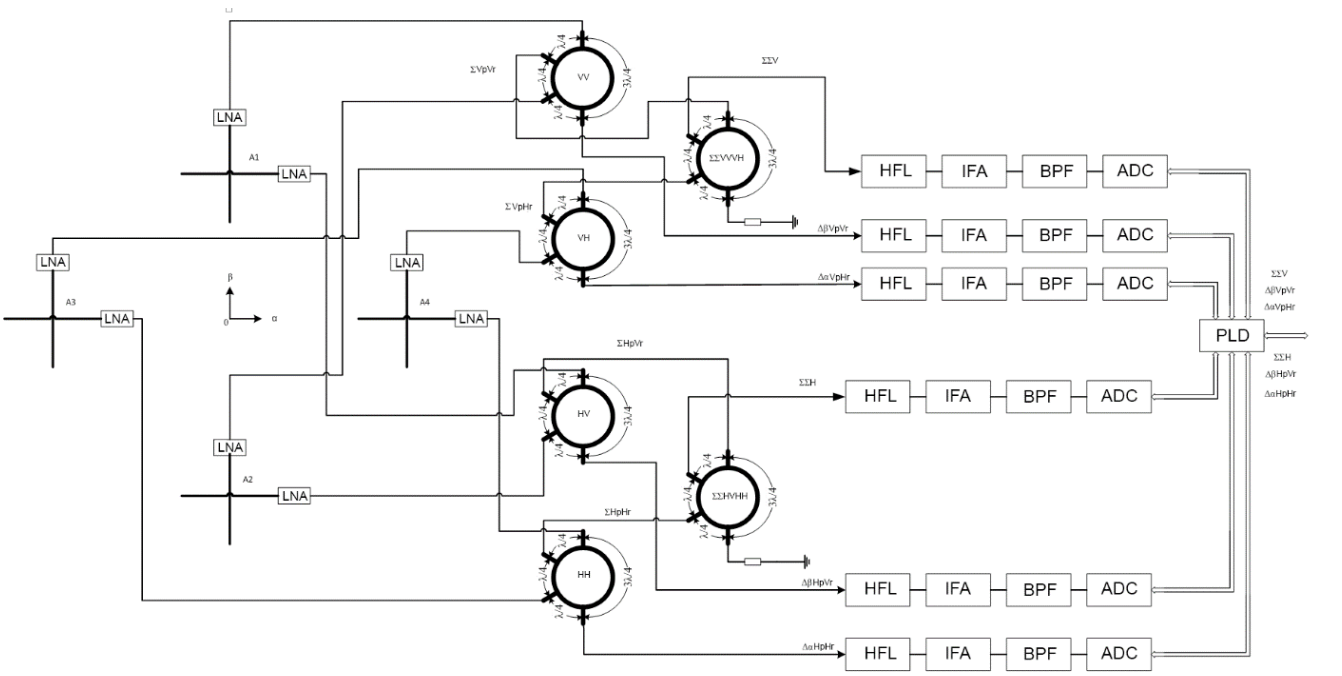

Fig. 5. Schematic diagram of the analogue forming and digital reception of sum and difference signals of plane-linear polarization from the B-529 antenna array

where k is the Bolzmann constant, t° is LNA front end noise temperature, ΔF is LNA band, the bandwidth filter (BWF) is the key factor in determining the signal to noise ratio after the IF amplifier, after which she signal-to-noise ratio equals

$$\left(\frac{P_{s\Sigma}}{P_{n\Sigma}}\right)_{\mathrm{A}} = 4 \cdot \frac{P_S}{P_n} \cdot \frac{\Delta F_{LNA}}{\Delta F_{IFA}}, \tag{12}$$

where ΔF_{IFA} is the bandwidth of the BWF after the IFA. When comparing the analog and the digital methods of summing of the signals from the B-529 antenna array, the signal ratios are the relevant factor, therefore the propagation ratio at the receive path can be assumed to be equal to 1.

The schematic diagram in Figure 6 shows the digital process of signal combining.

Using the digital signal processing method suggests having 8 independent signal channels with vertical and horizontal polarization from each of the four antennas of the B-529. Each of the channels consists of a high-frequency line, an intermediate frequency amplifier (IFA), a band-pass filter (BPF) and an analog to digital converter (ADC).

The digitized signals are fed to the programmable logic device (PLD), which processes them. The signals from the four antennas directed on the object, are as before phased by adjusting the length of the cable from the LNA to the HFL, identical to the analog processing. The hybrid rings are removed from the scheme. Their functions of forming of the sum and difference signals are performed by the PLD.

In this case, when the antenna is directed at the object, in-phase signals for the same polarization (vertical and horizontal) are fed into the ADC. In the PLD the signals with the same polarization are summed by amplitude. With a signal power of P_s and an amplitude of $\sqrt{2R}$ s the amplitude of the resulting signal will be equal $4\sqrt{2R}$ s and the power will be $16P_s$. The noise signals are summed by power, therefore, with a noise power in one of the channels of P_s at the LNA input, the noise power at the output of the band-pass filter of the IFA it will equal $P_s(\Delta F_{IFA}/\Delta F_{LFA})$, and after summing of the four signals it becomes $4P_s(\Delta F_{IFA}/\Delta F_{LFA})$. Therefore, the signal-to-noise ratio for the linear polarization after the summing of the signals from the four antennas equals

$$\left(\frac{P_{s\Sigma}}{P_{n\Sigma}}\right)_{D} = 4 \cdot \frac{P_S}{P_n} \cdot \frac{\Delta F_{LNA}}{\Delta F_{IFA}} \tag{13}$$

From (2) and (3) it is apparent that with the antenna array perfectly directed on the source of the signal will have equal signal-to-noise ratio for both digital and analog processing:

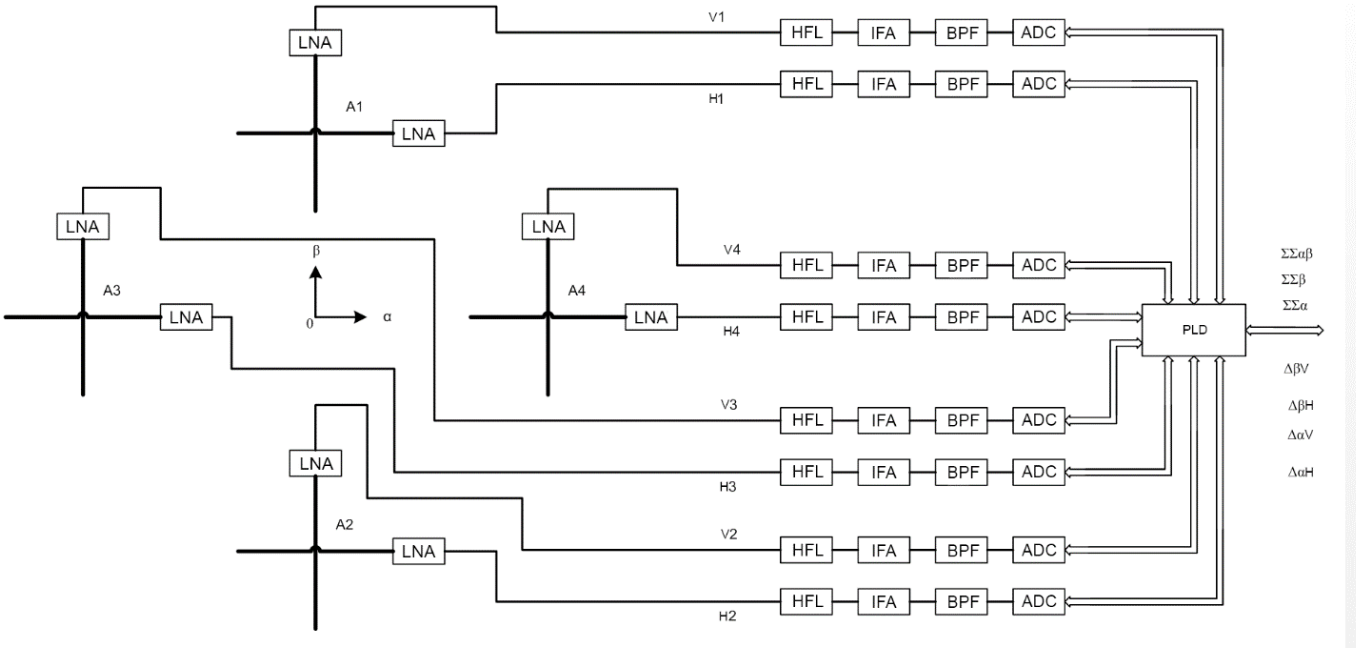

Fig. 6. Schematic diagram of digital signal processing RTS-9

$$\frac{\left(\frac{P_{s\Sigma}}{P_{n\Sigma}}\right)_D}{\left(\frac{P_{s\Sigma}}{P_{n\Sigma}}\right)_A} = 1 \tag{14}$$

Since the total antenna area of the four 6-meter dish antennas of the B-529 system is equal to the total area of the 12-meter TNA-57 antenna, the sum data signal range of the properly phased B-527 antenna array should be commeasurable with the range of the TNA-57 antenna.

Let us compare the characteristics of the analog and digital autotracking of the B-529 antenna array. It is imperative to consider here an example of an inaccurate antenna array positioning on the source of the radio signal.

If the signal power of a single antenna of the antenna array perfectly directed on a spacecraft is P_s, then the signal power from a signal antenna at the difference output port of the hybrid ring is $P_s/2$ with an amplitude of

$$U_m = \sqrt{RP_s} \tag{15}$$

The amplitude of a signal from a single antenna, deviated by Θ angle, equals

$$U_{m\Theta} = U_m \cdot F(\Theta), \tag{16}$$

where F(Θ) is the normalized pattern of a single parabolic antenna of the B-529 antenna array, which for the estimated calculations can be approximated as

$$F(\Theta) = \exp[-a \cdot (\Theta/\Theta_{0.5})^2] \tag{17}$$

Here $\Theta_{0.5}$ is half the width of the radiation pattern at 0.5 of the signal power and 0.707 of the signal amplitude and a = 0,346574.

With simultaneous deviation from the direction to the object by ΔΨ angle in elevation and by ΔΦ angle in azimuth the resulting deviation angle Θ is calculated from the relation for the angle between the vector of the radiation pattern and the vetor of the direction to the SC:

$$\cos\Theta = \cos(\alpha_{RP}) \cdot \cos(\alpha_{SC}) + \cos(\beta_{RP}) \cdot \cos(\beta_{SC}) + \\ + \cos(\gamma_{RP}) \cdot \cos(\gamma_{SC}), \tag{18}$$

where, in accordance with (2) and (7):

$$\cos(\alpha_{RP}) = \cos(\Phi_{SC} + \Delta\Phi) \cdot \sin(\Psi_{SC} + \Delta\Psi); \tag{19}$$

$$\cos(\beta_{RP}) = \cos(\Phi_{SC} + \Delta\Phi) \cdot \cos(\Psi_{SC} + \Delta\Psi); \tag{20}$$

$$\cos(\gamma_{RP}) = \sin(\Phi_{SC} + \Delta\Phi); \tag{21}$$

Hence,

$$\Theta = \arccos(\cos\Theta) \tag{22}$$

With the deviation of the axis of the phased antenna array from the direction to the SC, a phase shift $\Delta\varphi_{ij}$ occurs between the signals received by the antennas Ai and Aj. The phase shift is computed using the formulas (1) to (10).

The vector diagram for the difference signal of the antennas is presented on Fig. 7.

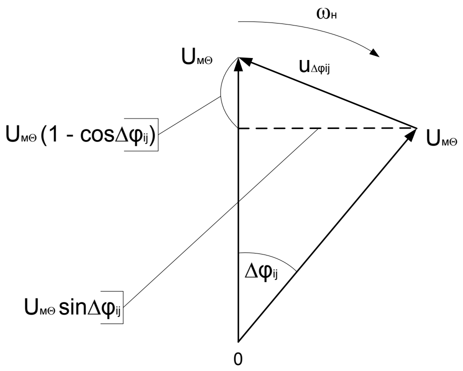

Fig. 7. The vector diagram for the difference signal of the antennas

When the antenna array is deviated from the object by ΔΨ in tilt, and by ΔΦ in azimuth, with the Ai and Aj antenna signals shifted in phase by Δφij, the modulus of the amplitude of the difference signal UΔφij in accordance with the vector diagram in Fig. 9 is calculated by the obvious formula:

$$U_{\Delta\varphi ij} = \sqrt{\begin{matrix}(U_{\text{м}\Theta} - U_{\text{м}\Theta} \cdot cos\Delta\varphi ij)^2 + \\ +(U_{\text{м}\Theta} \cdot sin\Delta\varphi ij)^2\end{matrix}}$$

Whence

$$U_{\Delta\varphi ij} = U_{\text{м}\Theta}\sqrt{2(1 - cos\Delta\varphi ij)} \tag{23}$$

The power of the difference signal of the Ai and Aj antennas after the analog subtraction, taking (15) and (16) into account, equals:

$$\begin{aligned} P_{\text{c}\Delta ij\text{A}} &= \frac{U^2_{\Delta\varphi ij}}{2R} = \frac{U^2_{\text{м}\Theta} \cdot 2(1 - cos\Delta\varphi ij)}{2R} = \\ &= \frac{U^2_{\text{м}} \cdot \left(F(\Theta)\right)^2 (1 - cos\Delta\varphi ij)}{R} = \\ &= \frac{R \cdot P_{\text{c}} \cdot \left(F(\Theta)\right)^2 (1 - cos\Delta\varphi ij)}{R} = \\ &= P_{\text{c}} \cdot \left(F(\Theta)\right)^2 (1 - cos\Delta\varphi ij) \end{aligned} \tag{24}$$

As mentioned before, the noise with a power of P_n from each of the two single antennas at the sum and difference output ports of the hybrid ring, add up to a single power of P_n. Therefore, the signal-to-noise power ratio for the difference signals from the Ai and Aj antennas deviated from the direction to the SC is calculated by the formula:

$$\frac{P_{\text{s}\Delta ij}}{P_{\text{n}\Delta ij}} = \frac{P_{\text{s}}}{P_{\text{n}}} \cdot \left(F(\Theta)\right)^2 (1 - cos\Delta\varphi ij) \tag{25}$$

After the BPF of the IFA, the signal-to-noise power ratio for the difference signals from the Ai and Aj antennas deviated from the direction to the SC becomes equal to:

$$\left(\frac{P_{\text{s}\Delta ij}}{P_{\text{n}\Delta ij}}\right)_{\text{BPF IFA}} = \frac{P_{\text{s}}}{P_{\text{n}}} \cdot \frac{\Delta F_{\text{LNA}}}{\Delta F_{\text{IFA}}} \cdot \left(F(\Theta)\right)^2 \cdot \\ \cdot (1 - cos\Delta\varphi ij) \tag{26}$$

Whence, the normalized values for the signal-to-noise ratio of the difference signal in relation to the signal-to-noise ratio of the signal from a single antenna after the BPF of the IFA with the analog subtraction:

$$\frac{\left(\frac{P_{\text{s}\Delta ij}}{P_{\text{n}\Delta ij}}\right)_{\text{BPF IFA}}}{\left(\frac{P_{\text{s}}}{P_{\text{n}}} \cdot \frac{\Delta F_{\text{LNA}}}{\Delta F_{\text{IFA}}}\right)} = \\ = (\text{F}(\Theta))^2 (1 - cos\Delta\varphi ij) \tag{27}$$

With the digital forming of the difference signal the signal-to-noise ratio, that equals P_s/P_n, remains the same up to the BPF of the IFA, and after the filter it equals $(P_s/P_n)(\Delta F_{LNA}/\Delta F_{IFA})$. With the digital subtraction the noise are summed by power, but, unlike with the hybrid bridge, the power of the signal isn't divided in two, therefore, instead of (15) we have:

$$U_c = \sqrt{2RPs} \tag{28}$$

Whence

$$U_s^2 = 2RP_s \tag{29}$$

The amplitude of the signal of a single antenna with the antenna array declined by Θ degrees equals

$$U_{s\Theta} = U_s \cdot F(\Theta), \tag{30}$$

By analogy with (16), (23) and (24) with regard of (28) – (30) with the digital signal processing method, before the BPF, a difference signal with twice the power is received:

$$P_{s\Delta ijD} = 2P_s \cdot \left(F(\Theta)\right)^2 (1 - cos\Delta\varphi ij) \tag{31}$$

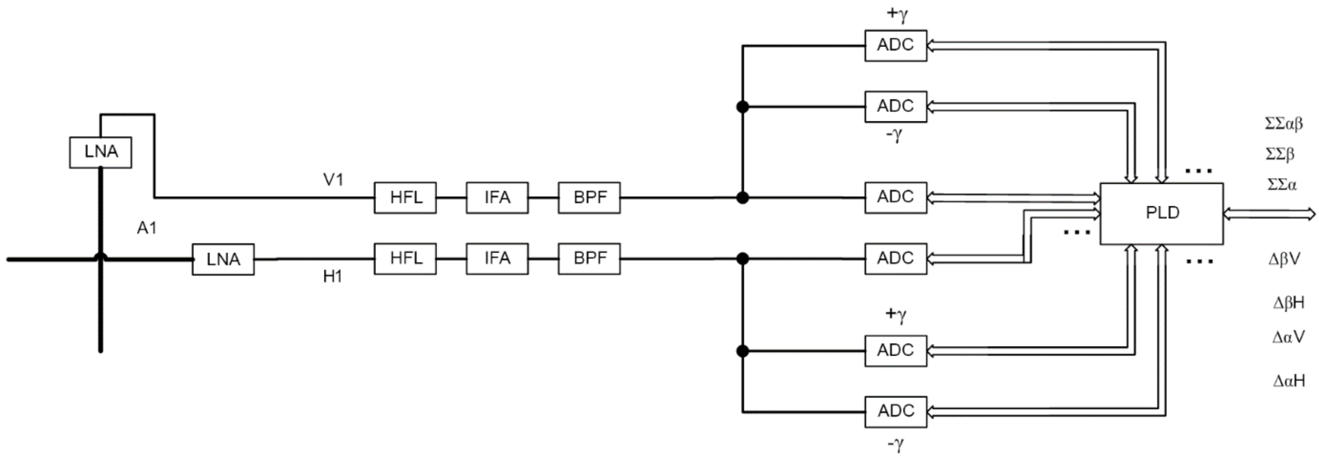

Fig. 8. Schematic diagram of digital signal processing of one of the four RTS-9 antennas for the difference auto-tracking of the sum signals.

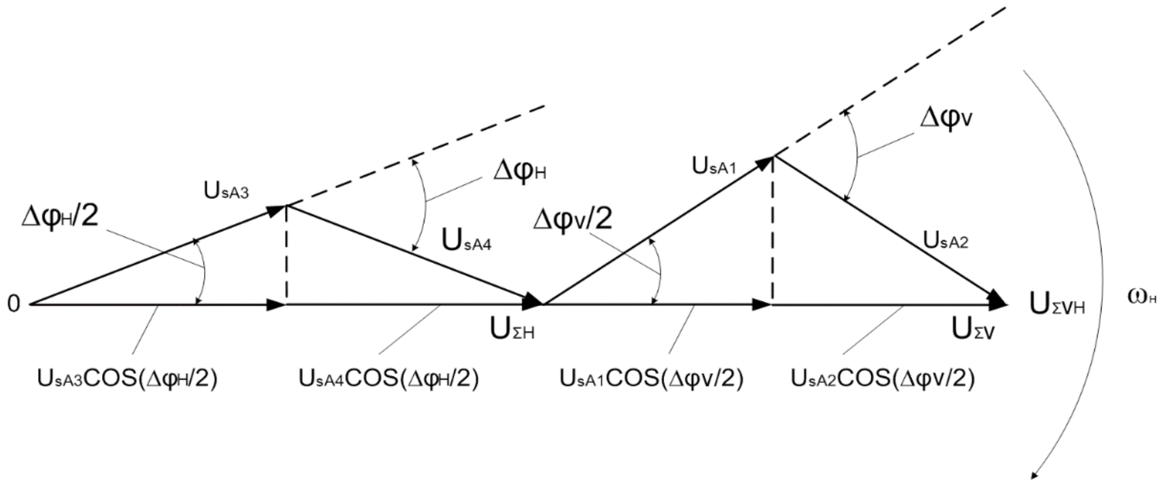

Fig. 9. Vector diagram for digital forming a sum signal of the vertical and horizontal rows of a deviated antenna array

However, the noise will also double in power. As a result, the same normalized values of signal-to-noise ratio of the difference signal are received, as compared to the signal-to-noise ratio of a single antenna signal after the BPF of the IFA, processed with the analog method:

$$\frac{\left(\frac{P_{s\Delta ij}}{P_{n\Delta ij}}\right)_{D\,BPF\,IFA}}{\left(\frac{P_s}{P_n}\cdot\frac{\Delta F_{LNA}}{\Delta F_{IFA}}\right)} = (\mathrm{F}(\Theta))^2(1-cos\Delta\varphi ij) \tag{32}$$

Therefore, the direct conversion of the method of difference signal forming from analog to digital processing will not improve the signal-to-noise ratio in the hypothetical case of a perfect analog processing. However, aside from being implemented with the perfect characteristics of the methods used for analog phasing, the digital processing enables new possibilities for the improvement of the signal-to noise ratio while forming a difference signal.

It is suggested that the forming of the difference guidance signal should be implemented for the sum signal from all of the four antennas of the B-529. For this purpose, it is suggested that the beam of the antenna array is oscillated horizontally and vertically and the difference signals of the combined beam of the array between the corresponding deviated positions are calculated. In other words, the beam central line is used. The schematic diagram of the suggested technical realization is presented in Figure 8.

This figure demonstrates the line of devices for a single antenna only. The schemes of the signal processing for the other 3 antennas are identical. Here, in addition to the ADC for the forming of the signal with vertical and horizontal polarization without a phase shift, two additional ADC are introduced to form a signal leading and lagging in phase by a certain angle μ, the selection of which will be discussed later. When the antenna is deviated from the direction to the object, alongside with the difference signals, upon which the principle of auto-tracking is based, being generated, there occurs a reduction in the power of the sum signal. The vector diagram for digital forming a sum signal of the vertical and horizontal rows of a deviated B-529 antenna array are presented in Figure 9.

The vector diagram in the Fig. 9 considers that in accordance with (1)–(10) the phase shifts of the signals of the A1 and A2 antennas related to the phase array center are equal in modulus and opposite in sign. The same is correct to the phase shifts in relation to the phase center of the antennas A3 and A4. This means that the pairwise sums of the signals from the A1, A2 antennas and A3, A4 antennas have the same phase, equal to the phase of the sum signal of all four antennas with any deviation of the radiation pattern axis from the direction to the spacecraft.

According to Figure 9, the amplitude of the digital sum signal is given by:

$$U_{\Sigma VH} = 2U_{s\Theta}\left[\cos\left(\frac{\Delta\varphi V}{2}\right) + \cos\left(\frac{\Delta\varphi H}{2}\right)\right] \tag{33}$$

The resulting power of the vertical and horizontal array rows:

$$P_{s\Sigma VH} = \frac{U_{\Sigma VH}^2}{2R} = \frac{\left[2U_{s\Theta}\left\{\cos\left(\frac{\Delta\varphi V}{2}\right) + \cos\left(\frac{\Delta\varphi H}{2}\right)\right\}\right]^2}{2R} =$$

$$= \frac{4 \cdot \cancel{2 \cdot R} \cdot P_s[F(\Theta)]^2\left[\cos\left(\frac{\Delta\varphi v}{2}\right) + \cos\left(\frac{\Delta\varphi h}{2}\right)\right]^2}{\cancel{2R}}, \tag{34}$$

$$P_{s\Sigma VH} = 4P_s[F(\Theta)]^2\left[\cos\left(\frac{\Delta\varphi V}{2}\right) + \cos\left(\frac{\Delta\varphi H}{2}\right)\right]^2$$

Since with the digital signal processing the noise in the four lines are independent, they are summed by power, after the BPF IFA:

$$P_{nVH} = 4P_n \cdot \frac{\Delta F_{IFA}}{\Delta F_{LNA}} \tag{35}$$

Hence the signal-to-noise ratio after the BPF IFA:

$$\frac{P_{s\Sigma VH}}{P_{nVH}} = \frac{P_s}{P_n} \cdot \frac{\Delta F_{LNA}}{\Delta F_{IFA}} \cdot$$
$$\cdot \left(F(\Theta)\right)^2 \left(\cos\left(\frac{\Delta\varphi V}{2}\right) + \cos\left(\frac{\Delta\varphi H}{2}\right)\right)^2 \tag{36}$$

its normalized value related to the signal-to-noise ratio for the signal from a single antenna after the BPF IFA equals:

$$\frac{\left(\frac{P_{s\Sigma VH}}{P_{nVH}}\right)}{\left(\frac{P_s}{P_n} \cdot \frac{\Delta F_{LNA}}{\Delta F_{IFA}}\right)} =$$
$$= (F(\Theta))^2 \left(\cos\left(\frac{\Delta\varphi V}{2}\right) + \cos\left(\frac{\Delta\varphi H}{2}\right)\right)^2 \tag{37}$$

Now let us evaluate the potential possibilities of the digital processing while forming a difference auto-tracking signal by the equisignal zone method based on the sum signal from all four antennas.

Let us examine the forming of the difference signal with the beam electronically declined in vertical direction. First, let us discuss the situation when the axis of the radiation pattern is deviated from the direction to the SC by ΔΨ in azimuth and by ΔΦ in elevation, presented on Figure 10.

In this situation the A1 and A3 antennas are extended towards the source of the signal. In other words, the B-529 is vertically declined towards the reduction of the elevation by ΔΦ angle, and horizontally by ΔΨ angle towards the increased azimuth angle.

Now we shall deflect the sum beam vertically towards the increasing elevation, introducing latency for the signal from the A1 antenna by the phase angle μ and lead for the signal from the A2 antenna by the same phase angle μ. This leads to some amount of compensation of the original declination of the antenna array towards the reduced elevation and the increase in the amplitude of the sum signal, as shown in Figure 10.

Now we shall decline the sum beam vertically towards the reducing elevation introducing the lead for the signal from the A1 antenna by the same phase angle μ and the lag for the A2 antenna by still the same phase angle μ. This leads to a greater deviation of the antenna array towards the reducing elevation in addition to the ΔΦ angle, and to even greater reduction of the amplitude of the sum signal.

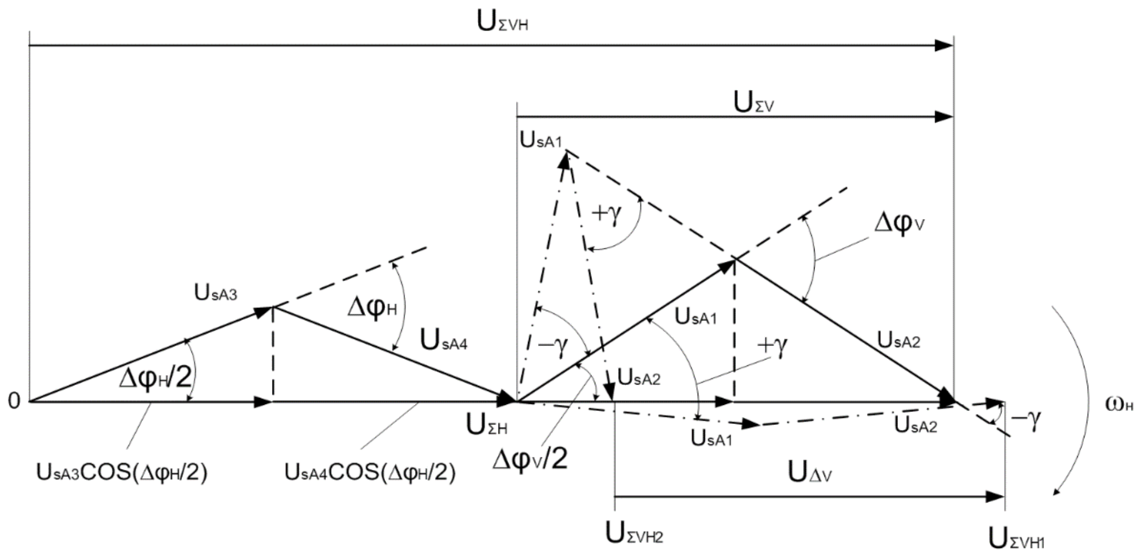

Fig. 10. Forming of the difference signal by the sum signal of the vertical row of the antenna array with a vertical deviation towards the reducing elevation.

Now we shall subtract the second sum signal from the first one to acquire the difference signal for the compensation of deviation of the antenna signal from the vertical direction to the object.

According to Fig. 10:

$$U_{\Sigma VH1} = U_{s\Theta}\left[2\cos\left(\frac{\Delta\varphi H}{2}\right) + \cos\left(-\frac{\Delta\varphi V}{2} + \mu\right) + \right.$$
$$\left. + \cos\left(+\frac{\Delta\varphi V}{2} - \mu\right)\right] \tag{38}$$

$$U_{\Sigma VH2} = U_{s\Theta}\left[2\cos\left(\frac{\Delta\varphi H}{2}\right) + \cos\left(-\frac{\Delta\varphi V}{2} - \mu\right) + \right.$$
$$\left. + \cos\left(+\frac{\Delta\varphi V}{2} + \mu\right)\right] \tag{39}$$

$$U_{\Delta V} = U_{\Sigma VH1} - U_{\Sigma VH2} =$$
$$= 2U_{s\Theta}\left[\cos\left(\frac{\Delta\varphi V}{2} - \mu\right) - \cos\left(\frac{\Delta\varphi V}{2} + \mu\right)\right] \tag{40}$$

The difference signal of the vertical guidance depends only on the sum signal of the two antennas of the vertical row. Therefore, in order to prevent the accumulation of additional noise from the orthogonal row of the array, when forming the vertical auto-tracking difference signal, it is advisable to use the sum signal only from the vertical antennas, and from the horizontal ones for the horizontal auto-tracking.

Figure 11 shows the procedure of forming of the difference vertical guidance signal with the antenna array declined towards the increasing elevation. The difference signal in this case is given by Formula (40), but it changes its sign to the opposite, since the angle of deviation ΔΦ changes its sign to the opposite, and, consequently, the phase shift Δφv changes its sign.

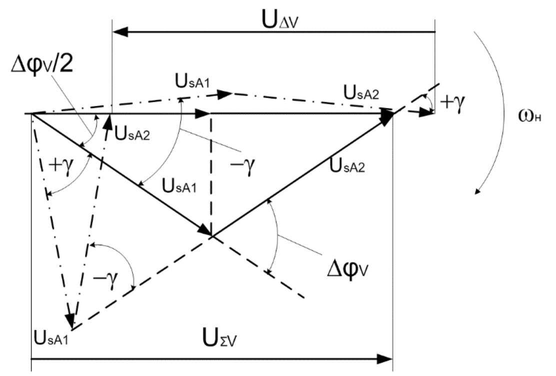

Fig. 11. Forming of the difference signal by the sum signal from the vertical row of the array with vertical deviation towards the increasing elevation.

Figure 12 shows, that without any vertical deviation of the antenna array, the difference signal equals 0 with any oscillating angle μ. Now let us evaluate the signal-to-noise ratio for the difference signal by the sum signal of the vertical row of the antenna array by the equisignal zone method.

From (40) the formula for the power of the difference signal by the sum signal of the vertical row of the array is deduced:

$$P_{s\Delta V} = 4P_s[F(\Theta)]^2\left[\cos\left(\frac{\Delta\varphi 12}{2} - \mu\right) - \cos\left(\frac{\Delta\varphi 12}{2} + \mu\right)\right]^2 \tag{41}$$

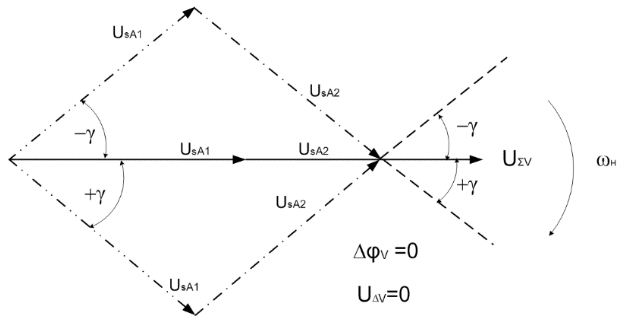

Figure 12. Forming of a zero difference signal by the sum signal of the vertical row of the array without vertical deviation – equisignal zone.

Since the noise from the LNA for the each of the "vertical" antennas twice independently affects the forming of the difference signal (when combining and subtracting), the total power of the noise after the BPF IFA equals:

$$P_{s\Delta V} = 4P_s \cdot \frac{\Delta F_{IFA}}{\Delta F_{LNA}} \tag{42}$$

From (41) and (42) the formula for the signal-to-noise ratio when forming the difference signal by the sum signal of the vertical row of the antennas by the equisignal zone method is derived:

$$\frac{P_{s\Delta V}}{P_{n\Delta V}} = \frac{P_s}{P_n} \cdot \frac{\Delta F_{LNA}}{\Delta F_{IFA}} \cdot$$
$$\cdot [F(\Theta)]^2 \left[\cos\left(\frac{\Delta\varphi 12}{2} - \mu\right) - \cos\left(\frac{\Delta\varphi 12}{2} + \mu\right)\right]^2 \tag{43}$$

Whence, the normalized value of the signal-to-noise ratio of the difference signal by the sum signal of the vertical row of the antenna array by the equisignal zone method, in relation to the signal-to-noise ratio of the input signal of a single antenna after the BPF IFA equals:

$$\frac{\left(\frac{P_{s\Delta V}}{P_{n\Delta V}}\right)}{\left(\frac{P_s}{P_n} \cdot \frac{\Delta F_{LNA}}{\Delta F_{IFA}}\right)} =$$
$$= (F(\Theta))^2 \left[\cos\left(\frac{\Delta\varphi 12}{2} - \mu\right) - \cos\left(\frac{\Delta\varphi 12}{2} + \mu\right)\right]^2 \tag{44}$$

The signal-to-noise ratio for the difference auto-tracking signals processed with digital and analog equisignal method is deduced by dividing (44) by (27):

$$\frac{\left(\frac{P_{\Delta V}}{P_{n\Delta V}}\right)_D}{\left(\frac{P_{\Delta V}}{P_{n\Delta V}}\right)_A} =$$
$$= \frac{\left[\cos\left(\frac{\Delta\varphi 12}{2} - \mu\right) - \cos\left(\frac{\Delta\varphi 12}{2} + \mu\right)\right]^2}{1 - cos\Delta\varphi 12} = 2 \cdot (sin\mu)^2 \tag{45}$$

Obviously, this equation attains a maximum at = 2 with μ=π/2 and sinμ=1.

Hence, with the deviation angle μ=π/2, the signal-to-noise ratio is two times higher when using the equisignal digital method as compared to the original analog method of acquisition of the difference auto-tracking signal on the hybrid ring couplers. This is correct providing two vertical and two horizontal antennas are used to form the difference signal. The downside is that three times as many ADC is required (24 rather than 8).

Notably, the design of the B-529 antenna array allows it to rotate the guidance axis by 45° and use the guidance method forming the sum and difference signals following the (A1 + A3) – (A4 + A2) scheme when the axis is declined by + 45° and the (A1+A4) – (A3+A2)) scheme when the axis is declined by – 45° (see Fig. 2).

Before we discuss this "diagonal" tracking method, it is important to note that according to (1) – (10) since the B-529 antenna array is symmetrical, the phase shifts are equal:

$$\Delta\varphi_{13} = \Delta\varphi_{42} \text{ and } \Delta\varphi_{14} = \Delta\varphi_{32} \tag{46}$$

Using the same method, as before, it is not hard to show that the normalized values of signal-to-noise ratio for the sum-difference signal (A1+A3)-(A4+A2) with analog processing in relation to the signal-to-noise-ratio of the signal from a single antenna after the BPF IFA when subtracted by the analog method:

$$\frac{\left(\frac{P_{s\Delta\varphi 13-42}}{P_{n\Delta\varphi 13-42}}\right)_{A\ BPF\ IFA}}{\left(\frac{P_s}{P_{Ш}} \cdot \frac{\Delta F_{LNA}}{\Delta F_{IFA}}\right)} =$$
$$= 2(F(\Theta))^2 \left[\cos\left(\frac{\Delta\varphi 13}{2}\right)\right]^2 (1 - cos\Delta\varphi 14) \tag{47}$$

With digital processing after BPF IFA the normalized signal-to-noise ratio is the same:

$$\frac{\left(\frac{\mathrm{P}_{s\Delta\varphi13-42}}{P_{n\Delta\varphi13-42}}\right)_{D\ BPF\ IFA}}{\left(\frac{P_s}{P_n}\cdot\frac{\Delta F_{LNA}}{\Delta F_{IFA}}\right)} = \tag{48}$$

$$= \left(\frac{\mathrm{P}_{s\Delta\varphi13-42}}{P_{n\Delta\varphi13-42}}\right)_{A\ BPF\ IFA} \bigg/ \left(\frac{P_s}{P_n}\cdot\frac{\Delta F_{LNA}}{\Delta F_{IFA}}\right)$$

Therefore, the digital signal processing method makes the perfect position control of a phased antenna array possible.

It is of interest to compare the signal-to-noise ratio (47) when guiding by the sum-difference signal from all four antennas to the signal-to-noise ratio (44) when guiding by the equisignal method for two antennas with an oscillating angle $\mu = \pi/2$:

$$\frac{4\cancel{(F(\Theta))^2}\left[sin\left(\frac{\Delta\varphi\nu}{2}\right)\right]^2}{\cancel{2\cdot(F(\Theta))^2}\left[\cos\left(\frac{\Delta\varphi13}{2}\right)\right]^2\cdot\cancel{2}\cdot\left[sin\left(\frac{\Delta\varphi14}{2}\right)\right]^2} = \tag{49}$$

$$= \frac{\left[sin\left(\frac{\Delta\varphi12}{2}\right)\right]^2}{\left[\cos\left(\frac{\Delta\varphi13}{2}\right)\right]^2\left[sin\left(\frac{\Delta\varphi14}{2}\right)\right]^2}$$

The numerator attains a maximum 1 at $\Delta\varphi_{12} = \pm\pi$. The denominator attains a maximum at $\Delta\varphi_{13} = 0$ or 2π и $\Delta\varphi_{14} = \pm\pi$, i.e. when there is no deviation along the A1-A3 antennas. Since the signal-to-noise ratio for the equisignal method for two antennas does not depend on the lateral deviation of the radiation pattern, and the signal-to-noise ratio for the sum-difference method from all four antennas of the array decreases with a lateral deviation of the pattern, then with identical angular deviation, the signal-to-noise ratio of the equisignal method is never lower than of the sum-difference signal from all four antennas.

Furthermore, a modification of the equisignal method for all four antennas is possible, that will double the signal-to-noise ratio than the sum-difference tracking method. The essence of the modification is to scan the radiation pattern of the array by a certain ν angle in both directions with a ± 45° inclination and extract the difference signal.

By the same method, it can be shown that the normalized signal-to-noise ratio of the difference signal from the four antennas by the equisignal method after the BPF IFA is given by:

$$\frac{\left(\frac{P_{s\Sigma1234\Delta\varphi14}}{P_{\mathrm{n}1234\Delta\varphi14}}\right)}{\left(\frac{P_{\mathrm{s}}}{P_{\mathrm{n}}}\cdot\frac{\Delta F_{\mathrm{LNA}}}{\Delta F_{\mathrm{IFA}}}\right)} =$$

$$= 2\cdot\left(F(\Theta)\right)^2\left[\cos\left(\frac{\Delta\varphi13}{2}\right)\right]^2\cdot \tag{50}$$

$$\cdot\left[\cos\left(\frac{\Delta\varphi14}{2}+\nu\right)-\cos\left(\frac{\Delta\varphi14}{2}-\nu\right)\right]^2$$

By dividing (67) by (47) let us compare the normalized signal-to-noise ratios for the difference signal (A1+A3) – (A4+A2) and the difference signal from the four antennas by the equisignal method:

$$\frac{\left(\frac{P_{s\Sigma1234\Delta\varphi14}}{P_{\mathrm{n}1234\Delta\varphi14}}\right)}{\left(\frac{\mathrm{P}_{s\Delta\varphi13-42}}{P_{\mathrm{n}\Delta\varphi13-42}}\right)} =$$

$$= 2\cdot\left(F(\Theta)\right)^2\left[\cos\left(\frac{\Delta\varphi13}{2}\right)\right]^2\left[\cos\left(\frac{\Delta\varphi14}{2}+\nu\right)-\right.$$

$$\left.-\cos\left(\frac{\Delta\varphi14}{2}-\nu\right)\right]^2\bigg/\ 2\left(F(\Theta)\right)^2\left[\cos\left(\frac{\Delta\varphi13}{2}\right)\right]^2\cdot \tag{51}$$

$$\cdot\ (1-cos\Delta\varphi14)\ = 2\cdot(sin\nu)^2$$

This expression, obviously, attains a maximum 2 at $\nu = \pi/2$, with $\sin\nu = 1$. Hence, by selecting the declination angle $\nu = \pi/2$, the signal-to-noise ratio of the equisignal digital method can be doubled in power, as compared to the original analog hybrid ring coupler auto-tracking method.

The results received are illustrated by the plots in Figures 13 – 21, showing the signal-to-noise ratios of the sum and difference signals for the analog and digital processing at a carrier frequency of 150 MHz at different sections of the radiation pattern.

Figures 13 – 16 feature the plots of signal-to-noise ratio against deviations in elevation and azimuth that illustrate the influence of the elevation angle of the target on the deviation in azimuth. With target elevation angles close to zenith, the azimuth antenna guidance is lost with any tracking method. This does not occur when error-based tracking in mutually perpendicular directions at an angle of 45° is employed, as shown in figures 17 – 20.

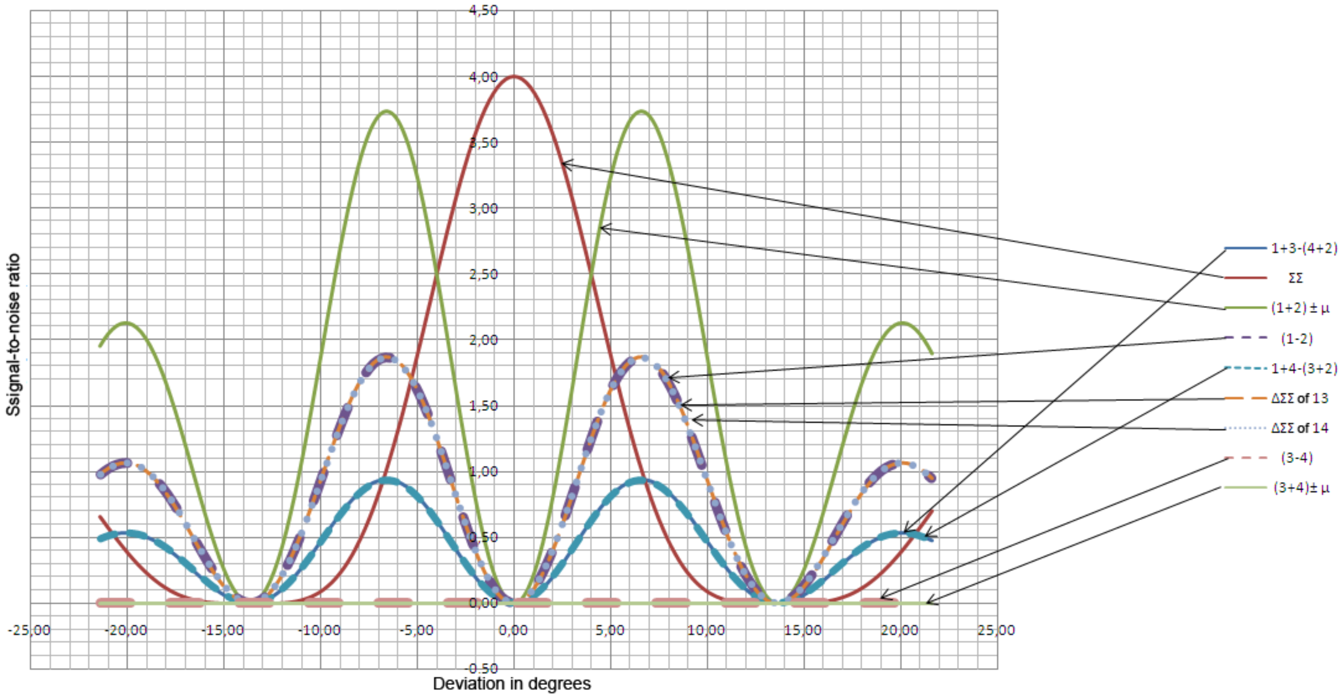

Fig. 13. Elevation error. Does not depend on azimuth or elevation.

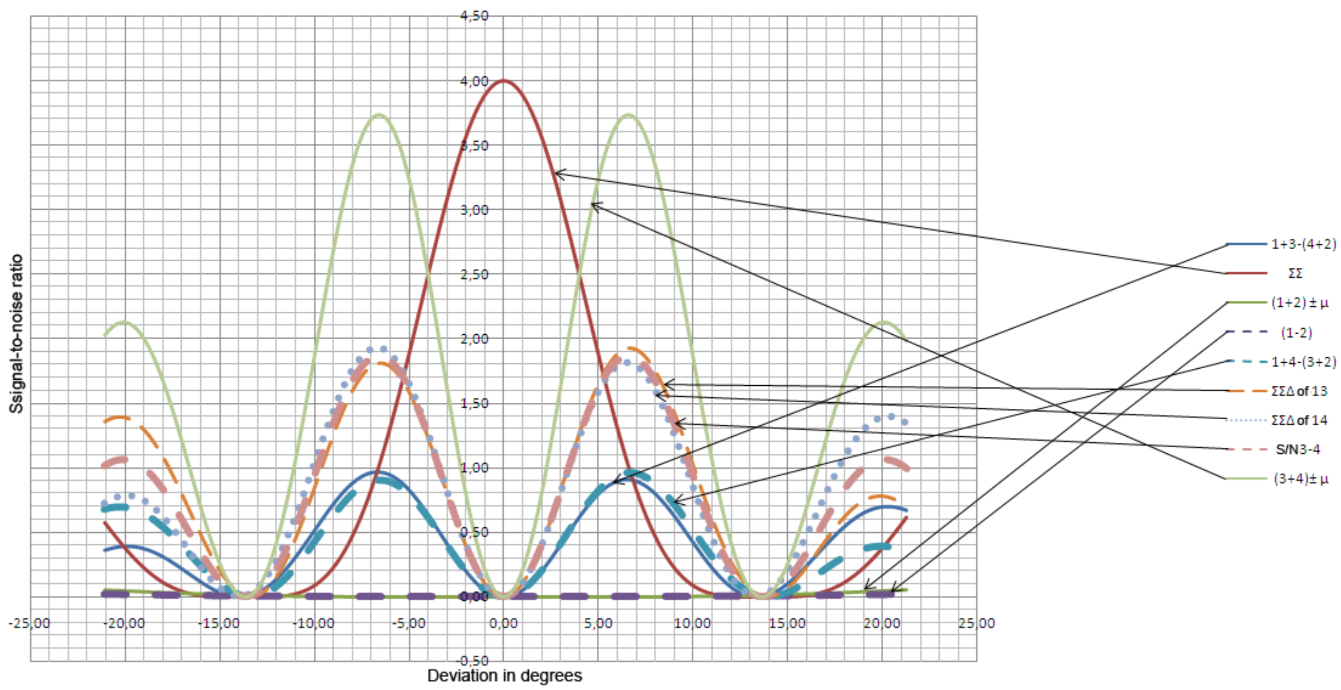

Fig. 14. Azimuth error. 10° elevation.

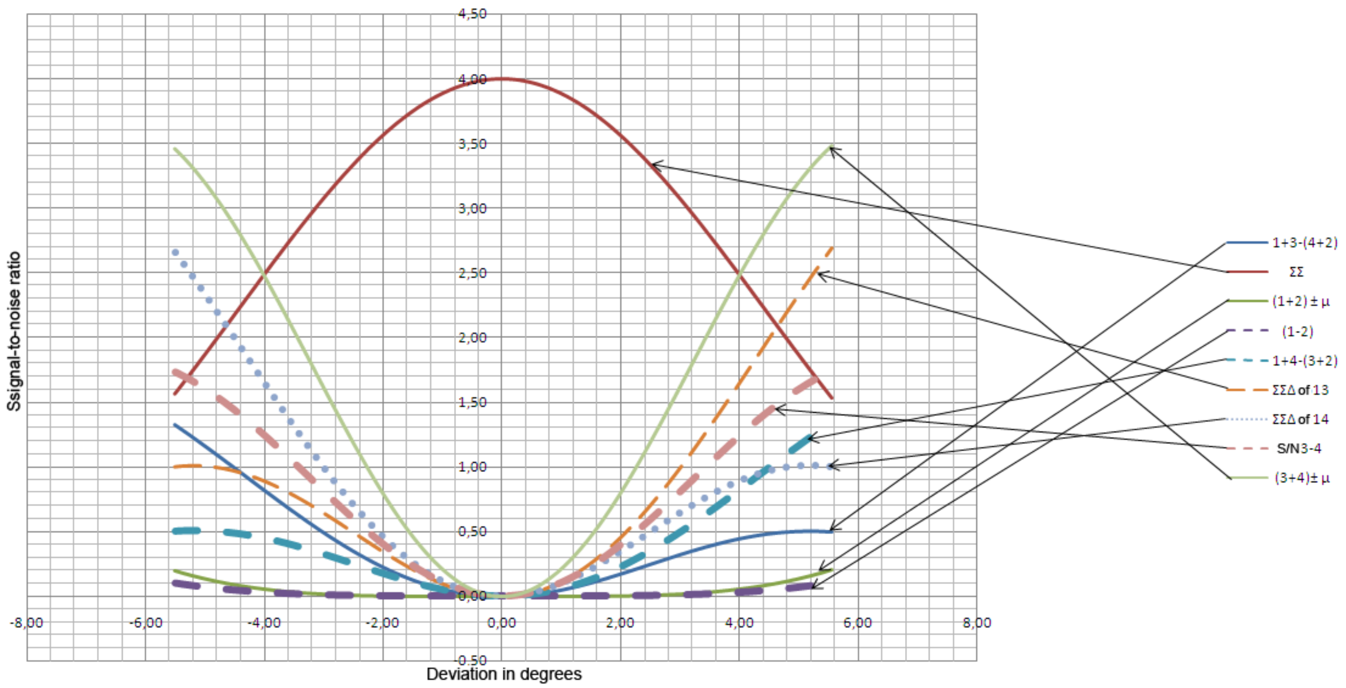

Fig. 15. Azimuth error. 75° elevation.

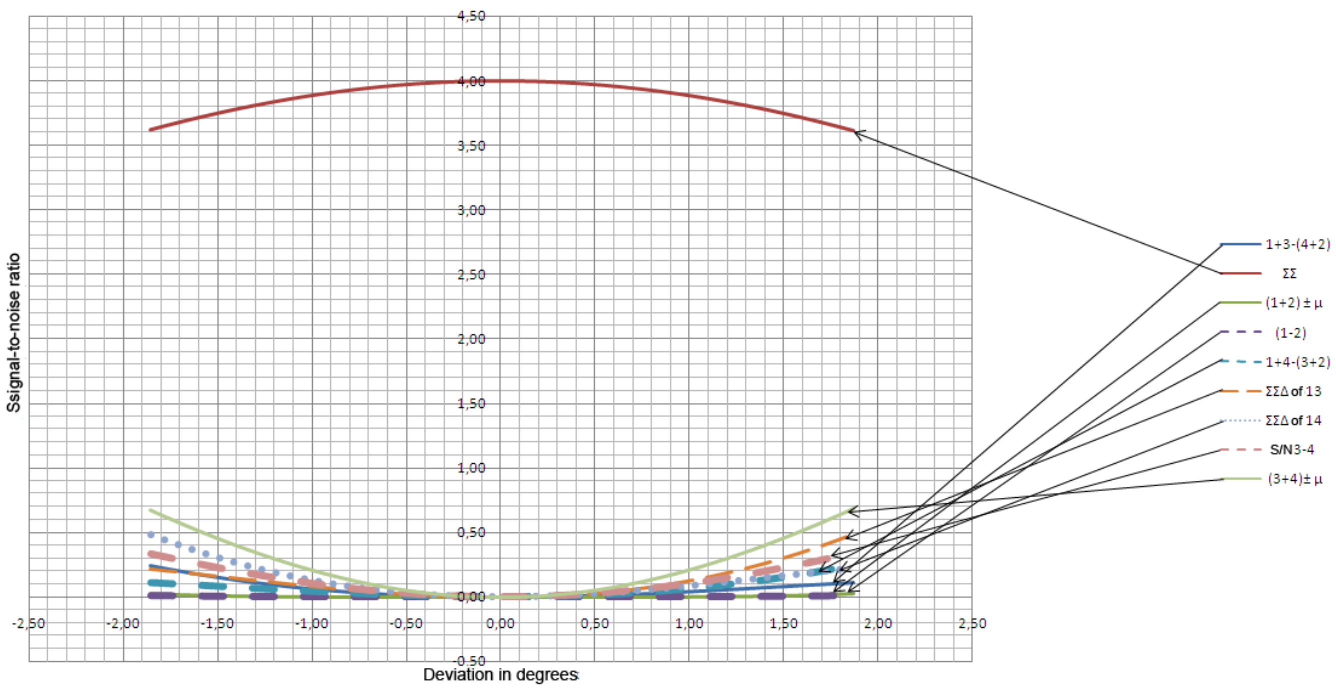

Fig. 16. Azimuth error. 85° elevation.

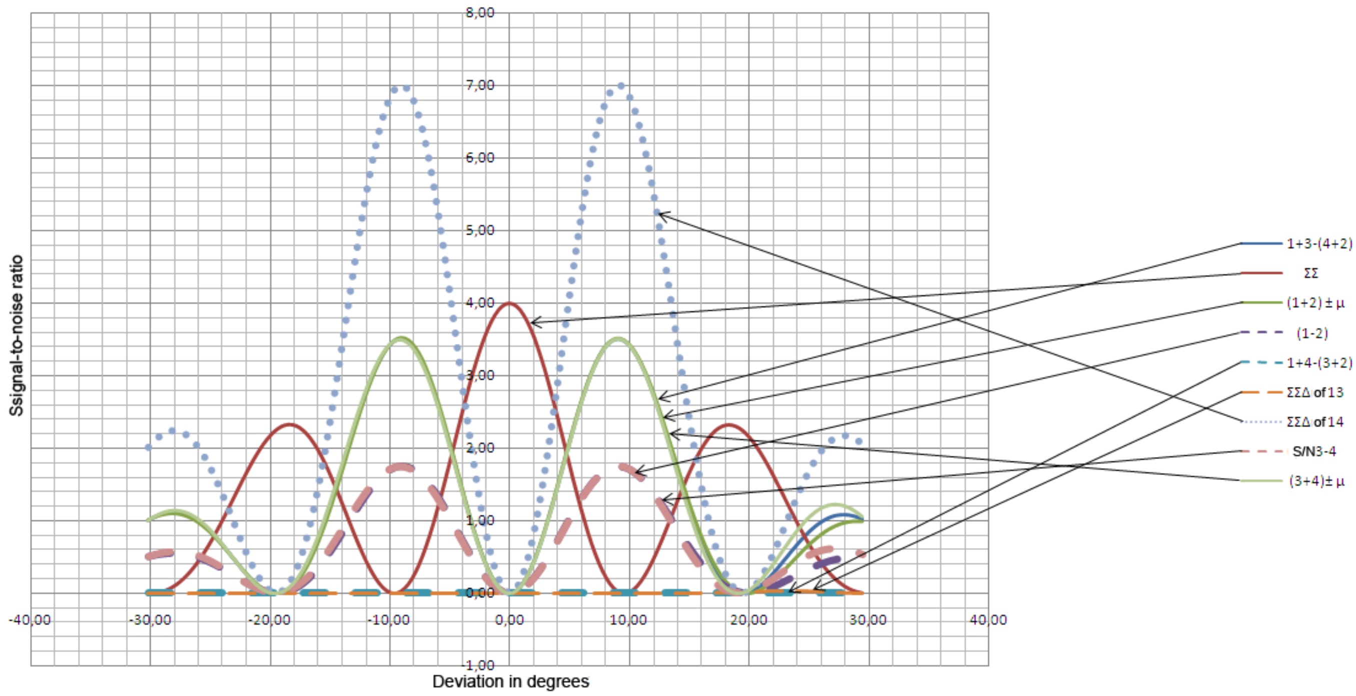

Fig. 17. Direction error for the A1-A4 antennas. 10° elevation. Does not depend on azimuth.

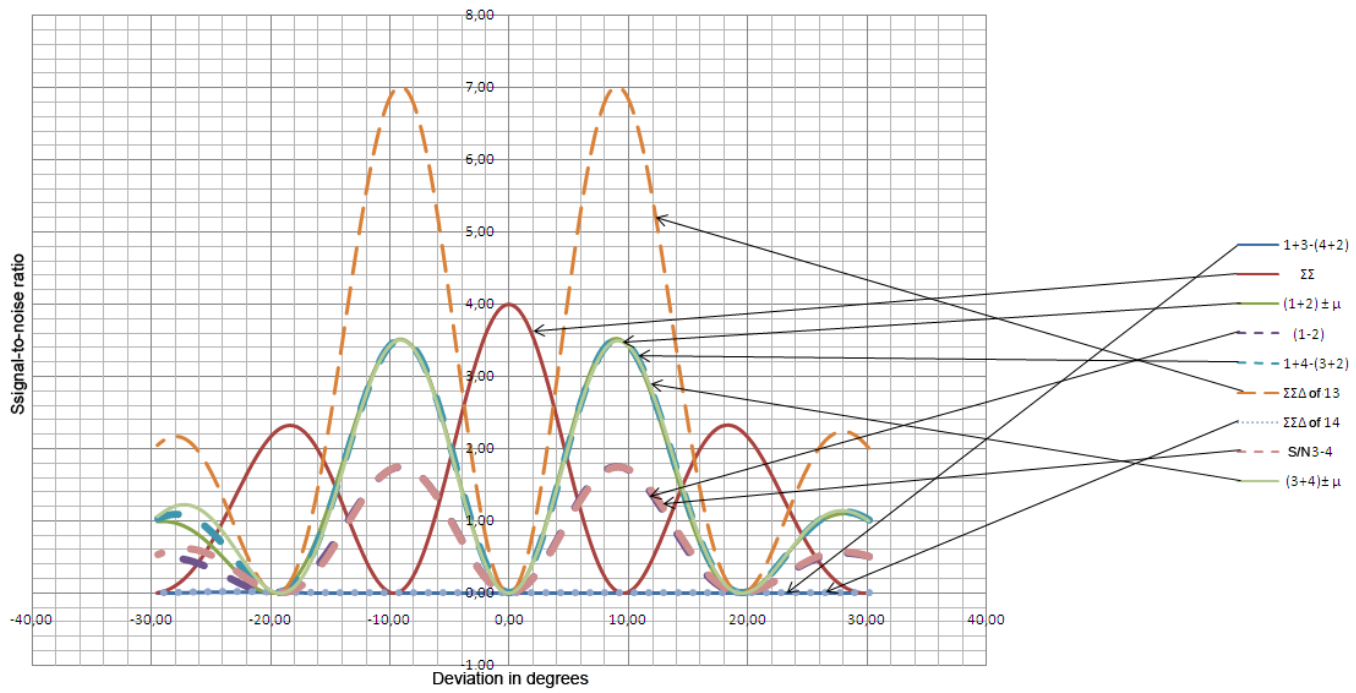

Fig. 18. Direction error for the A1-A3 antennas. 10° elevation.

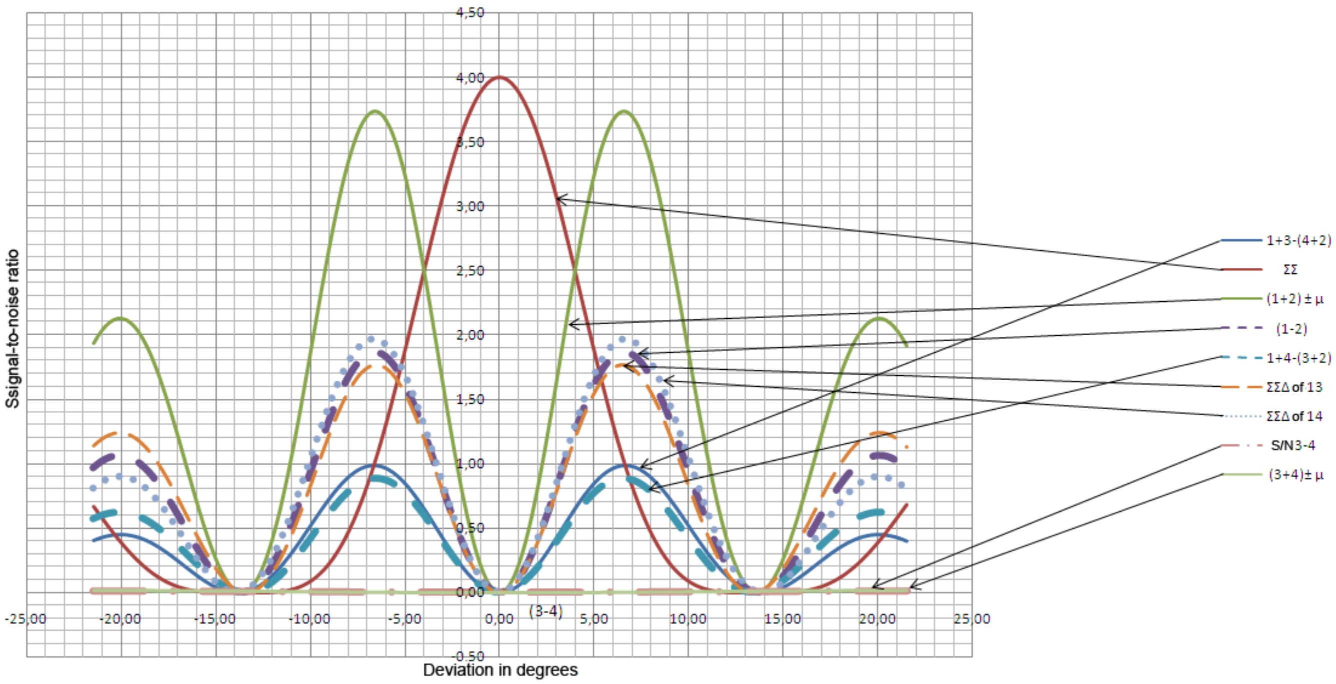

Fig. 19. Direction error for the A1-A4 antennas. 89° elevation.

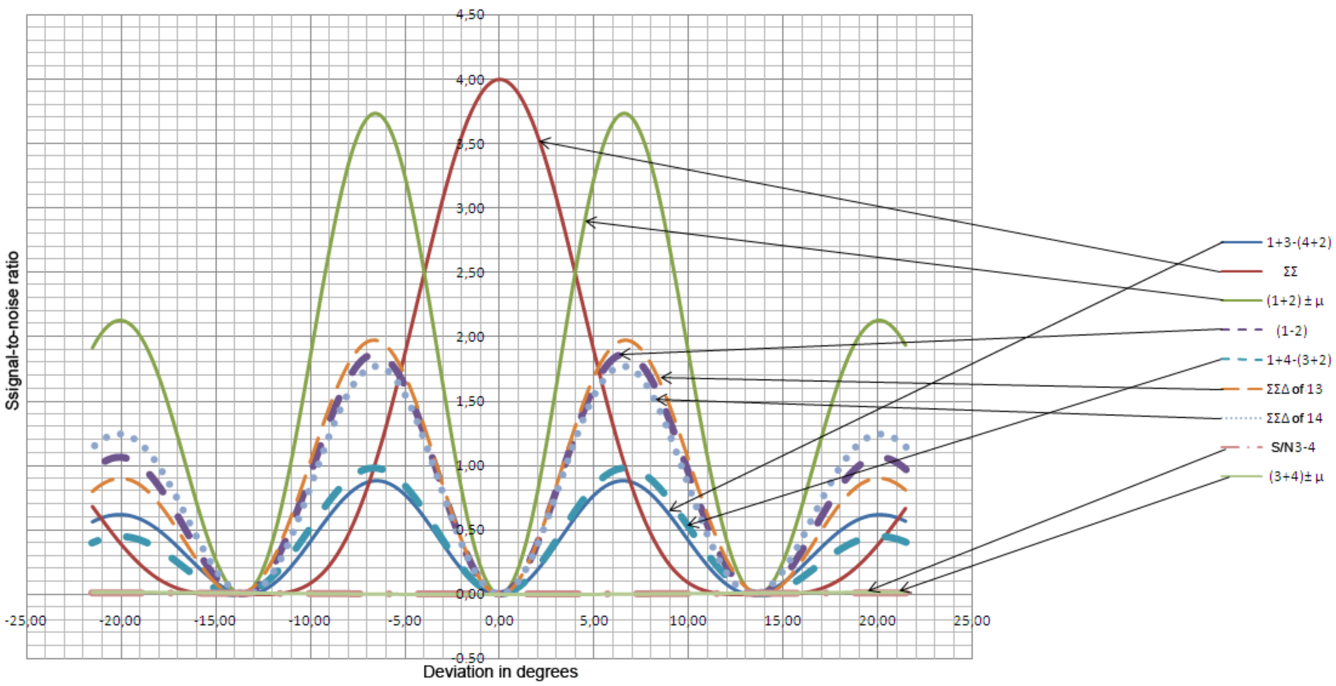

Fig. 20. Direction error for the A1-A3. 89° elevation.

As seen in Figures 19 – 20, the guidance by the difference signals in mutually perpendicular directions at an angle of 45° is not lost even in zenith, besides, the signal-to-noise ratio of the difference signal from all four antennas by the method of the equisignal zone is twice as high as the signal-to-noise ratio of the difference signal of the pairs of antennas angled at 45°, i.e. (A1+A3) – (A4+A2) and (A1+A4) – (A3+A2) signals. The downside is the complicated processing and angle control for the introduced phase shifts during the analog-to-digital conversion, since the phase shift by 90° when scanning the sum beam at 45° is determined by the phase shifts in azimuth and elevation, and those phase shifts consequently depend on the elevation of the target.

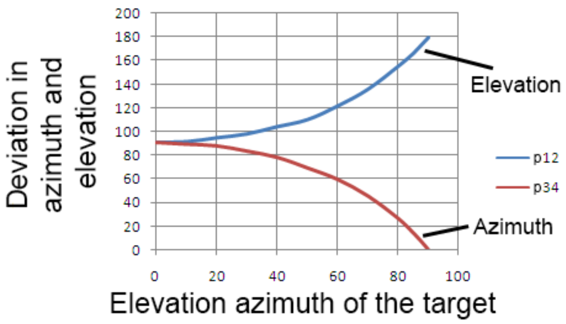

Fig. 21. Errors on the analog-to-digital converter in azimuth and elevation when scanning the radiation pattern by ± 90° at 45° angle.

Conclusion

1. Digital processing provides the same signal-to-noise ratio as an ideal analog processing, i.e. digital radio signal processing allows for perfect guidance of a phased antenna array for the information transmission sum signal from four antennas of the B-529 system.

2. For the difference signal, the equisignal autotracking method with digital processing when applied properly, can give a gain in the signal-to-noise power by 2 times compared the analog method in ideal conditions, thus increasing the range of the spacecraft-ground radio link in $\sqrt{2}$ times.

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