Technique for Hardware Residuals Evaluation of GNSS-Measurements of Pseudorange in Phase of the Carrier of the Onboard Navigation System of LEO Spacecraft

A.P. Fursov, *Cand. Sci. (Engineering), contact@spacecorp.ru*
Joint Stock Company "Russian Space Systems", Moscow, Russian Federation
A.A. Fursov, *contact@spacecorp.ru*
Joint Stock Company "Russian Space Systems", Moscow, Russian Federation
V.S. Vdovin, *Cand. Sci. (Engineering), contact@spacecorp.ru*
Joint Stock Company "Russian Space Systems", Moscow, Russian Federation
A.V. Zaichikov, *contact@spacecorp.ru*
Joint Stock Company "Russian Space Systems", Moscow, Russian Federation
Z.A. Pozyaeva, *contact@spacecorp.ru*
Joint Stock Company "Russian Space Systems", Moscow, Russian Federation

Abstract. When using the onboard satellite navigation equipment of LEO spacecraft, there arises a problem of getting an adequate evaluation of hardware errors of the equipment of satellite navigation by real results of navigation observations without a reference orbit or measurements. This article proposes a technique of a hardware error evaluation of pseudorange measurements of the carrier frequency phase received by the onboard equipment of satellite navigation installed on LEO spacecraft. A hardware error of the mentioned measurements is determined indirectly by the so-called double differences of the initial measurements, which make it possible to rule out the overwhelming portion of systematic errors.
The results of applying this technique on two particular selections (S1, S2) of measurements with durations of 5 hours each for one of the Russian spacecraft similar to the spacecraft of the JASON space system are presented. An evaluation of phase measurement accuracy was performed for radio signals in the frequency ranges L1, L2 of GLONASS navigation spacecraft.

Keywords: navigation systems, navigation measurements, evaluation of measurement accuracy, LEO spacecraft, hardware errors, double differences

Introduction

An assessment of errors of results of navigation measurements subject to random and unknown systematic errors in the absence of control measurements, at least, much more exact, is generally impossible or significantly becomes complicated. Under certain conditions, if it is possible to exclude systematic errors of measurements and find an adequate representation of the measured function, an assessment of a random ("noise-like") component of an error of results of measurements as a stationary process can be defined.

The offered technique is developed to assess a hardware error of results of measurements of a carrier phase received by the equipment of satellite navigation of LEO spacecraft using GNSS and GPS SC. Applying this technique and technology of its realization is considered in relation to one of domestic LEO spacecraft (further – GS SC).

The mentioned technique can be used for assessment of a hardware error of GNSS-measurements of a carrier phase in the equipment of satellite navigation of LEO spacecraft of a different purpose.

1. Technique to estimate errors of measurements of pseudorange by a carrier phase

Definition of dispersion of errors of any undisplaced measurements subject to random errors, generally comes down to definition of a residual vector of measurements which components are the differences of results of measurements and their true calculated values (the measured function). If the measured function is known and casual errors of measurements have a limited dispersion and are not displaced, then an undisplaced assessment of dispersion is the square of a norm of a residual vector divided into dimension of this vector minus the quantity of degrees of freedom of a residual vector.

In practice, results of measurements of a carrier phase do not meet the conditions mentioned above because of presence of the shifts caused by the influence of a set of various technical factors and effects assossiated with conditions of signal distribution.

The positive result can be received when using instead of initial results of measurements of a carrier phase of differential measurements as a result of linear transformations of initial measurements, in particular, of the so-called double differences (DD) of a carrier phase [1].

The double differences calculated on simultaneous onboard measurements of a carrier phase of LEO spacecraft and ground measurements of a carrier phase (from ground measuring stations), have almost no systematic errors caused by the influence of the onboard equipment.

The essence of the technique offered in this paper consists in receiving an assessment of a hardware error of initial measurements of a carrier phase indirectly – according to evaluation of errors of DD of a carrier phase. If the assumption is admissible that initial phase measurements are equally accurate, then casual errors of differential measurements, as a rule, represent the errors typical of a stationary process. Further, if dispersion of errors for DD of a carrier phase is defined, then dispersion for any private component of its measurements of a carrier phase can be calculated taking into account the weight fraction considering the used combinations of initial measurements in the form of an amplification factor of the corresponding measurement in differential measurement. As a result, if $\boldsymbol{\delta}_{\mathbf{r}}$ is the assessment of an RMS error for DD of a carrier phase [1], then $\boldsymbol{\delta}$ is the assessment of an RMS error of the components of its measurements of a carrier phase is obtained by division of $\boldsymbol{\delta}_{\mathbf{r}}$ into the amplification factor $\boldsymbol{\gamma}$.

Let us suggest that the measured function **u** and **n** of the fixed values of this function $\boldsymbol{x_1, x_2, \dots, x_n}$ of the final interval of a selection are interconnected by the following ratios: $\boldsymbol{u} = \boldsymbol{f}(\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n})$, $\boldsymbol{u} = \boldsymbol{u}^* + \boldsymbol{\varepsilon_u}$, $x_i = x_i^* + \varepsilon_i$, where $\boldsymbol{u}^*$, $\boldsymbol{x_i^*}$ are the true values of the variables, $\boldsymbol{\varepsilon_u}$, $\boldsymbol{\varepsilon_i}$ are the random unbiased errors.

Taking into account the chosen assumptions

$$\boldsymbol{\varepsilon_u^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 \varepsilon_i^2}$$

and initial phase measurements, pseudorange on the i-th frequency are the following:

$$\mathbf{L_i = D + I_i + \sum_j \Delta D_j + \varepsilon_i}$$

where $\boldsymbol{L_i}$ is the result of measurement of a carrier phase on the *i*-th frequency; D is the true range between the satellite and receiver; $\boldsymbol{I_i}$ is the correction connected with signal distribution in the Earth ionosphere; $\sum_j \boldsymbol{\Delta D_j}$ is the sum of other corrections that are not examined in detail in this case; $\boldsymbol{\varepsilon_i}$ is the random noises of a signal on the i-th frequency.

In the expression for $\boldsymbol{L_i}$, the value $\sum_j \boldsymbol{\Delta D_j}$ is the sum of systematic and slowly changing errors of phase measurements, which are eliminated when forming DD. An unremovable part of these errors is eliminated at the optimal approximation of the measurement results in the way when a dominating error of polynomial residual are only hardware errors.

An ionosphere-free combination of phase measurements on the frequencies *f1* and *f2* [1] is the following:

$$\boldsymbol{L_{IF}} = \frac{f_1^2}{f_1^2 - f_2^2}\boldsymbol{L_1} - \frac{f_2^2}{f_1^2 - f_2^2}\boldsymbol{L_2} = \boldsymbol{D} + \sum_j \boldsymbol{\Delta D_j^*} + \boldsymbol{\varepsilon_{IF}}$$

where the $\boldsymbol{\varepsilon_{iF}}$ value is calculated buy the formula:

$$\boldsymbol{\varepsilon_{IF}} = \sqrt{\left(\frac{f_1^2}{f_1^2 - f_2^2}\right)^2 \varepsilon_1^2 + \left(\frac{f_2^2}{f_1^2 - f_2^2}\right)^2 \varepsilon_2^2} = \frac{\sqrt{f_1^4\varepsilon_1^2 + f_2^4\varepsilon_2^2}}{f_1^2 - f_2^2}.$$

Considering that $\frac{f_1}{f_2} = \boldsymbol{\gamma}$, we obtain the formula:

$$\boldsymbol{\varepsilon_{IF}} = \frac{\sqrt{\gamma^4\varepsilon_1^2 + \varepsilon_2^2}}{\gamma^2 - 1}.$$

For a geometrically-free combination of phase measurements on the frequencies *f1* and *f2*, there is a following ratio:

$$\boldsymbol{L_{GF}} = \boldsymbol{L_1} - \boldsymbol{L_2} = \boldsymbol{I_{GF}} + \boldsymbol{\lambda_1 n_1} - \boldsymbol{\lambda_2 n_2} + \boldsymbol{\varepsilon_{GF}}$$

where $\boldsymbol{\varepsilon_{GF}} = \sqrt{\varepsilon_1^2 + \varepsilon_2^2}$.

A double difference of ionosphere-free combinations of phase measurements can be presented this way:

$$\boldsymbol{L_{IF}} = (\boldsymbol{\nabla\Delta L_{IF11}} - \boldsymbol{L_{IF12}}) - (\boldsymbol{L_{IF21}} - \boldsymbol{L_{IF22}}) = (\boldsymbol{D_{11}} - \boldsymbol{D_{12}}) - (\boldsymbol{D_{21}} - \boldsymbol{D_{22}}) + \sum_j \boldsymbol{\Delta D_j^{**}} + \boldsymbol{\varepsilon_{\nabla\Delta}}$$

where

$$\boldsymbol{\varepsilon_{\nabla\Delta}} = \sqrt{\varepsilon_{IF}^2 + \varepsilon_{IF}^2 + \varepsilon_{IF}^2 + \varepsilon_{IF}^2} = 2\boldsymbol{\varepsilon_{IF}} = 2\frac{\sqrt{\gamma^4\varepsilon_1^2 + \varepsilon_2^2}}{\gamma^2 - 1}.$$

Let us consider two cases:

1. A random noise of phase measurements on the *f1* and *f2* frequencies is the same by the value, i.e., $\boldsymbol{\varepsilon_1} = \boldsymbol{\varepsilon_2}$, then for $\boldsymbol{\varepsilon_{\nabla\Delta}}$ and $\boldsymbol{\varepsilon_{GF}}$ we receive:

$$\boldsymbol{\varepsilon_{\nabla\Delta}} = 2\frac{\sqrt{\gamma^4 + 1}}{\gamma^2 - 1}\boldsymbol{\varepsilon_1}$$

$$\boldsymbol{\varepsilon_{GF}} = \sqrt{2}\boldsymbol{\varepsilon_1}.$$

Taking into account that for GLONASS navigation SC $\boldsymbol{\gamma} = 9/7$ [1], we receive:

$$\boldsymbol{\varepsilon_{\nabla\Delta}} \simeq 5.917, \quad \boldsymbol{\varepsilon_{GF}} \simeq 1.414\boldsymbol{\varepsilon_1}.$$

2. A random noise of phase measurements is directly proportional to the wavelength of the signal $\boldsymbol{\varepsilon_i} = \boldsymbol{\alpha\lambda_i}$, i.e., $\boldsymbol{\varepsilon_2}/\boldsymbol{\varepsilon_1} = \boldsymbol{\gamma}$, then for $\boldsymbol{\varepsilon_{\nabla\Delta}}$ and $\boldsymbol{\varepsilon_{GF}}$ we receive:

$$\boldsymbol{\varepsilon_{\nabla\Delta}} = 2\gamma\frac{\sqrt{\gamma^2 + 1}}{\gamma^2 - 1}\boldsymbol{\varepsilon_1}$$

$$\boldsymbol{\varepsilon_{GF}} = \sqrt{1 + \gamma^2}\boldsymbol{\varepsilon_1}.$$

Considering that for GLONASS navigation SC $\boldsymbol{\gamma} = 9/7$ [1], we receive:

$$\boldsymbol{\varepsilon_{\nabla\Delta}} \simeq 6.413\boldsymbol{\varepsilon_1}, \quad \boldsymbol{\varepsilon_{GF}} \simeq 1.629\boldsymbol{\varepsilon_1}.$$

Amplification factors of a random noise for different combinations of phase measurements are given in Table 1.

Table 1. The amplification factors $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$ of a random noise for different combinations of phase carrier in relation to the noise at the L1 frequency.

	k_1 (at $\varepsilon_1 = \varepsilon_2$)	k_2(at $\varepsilon_2/\varepsilon_1 = \gamma$)
ε_1	1	1
ε_2	1	1.286
ε_{GF}	1.414	1.629
ε_{IF}	2.958	3.206
$\nabla\Delta\varepsilon_{IF}$	5.917	6.413

The results show that the estimation of an RMS error of carrier phase measurements can be calculated implicitly based on the estimation of an RMS error of the carrier phase measurements for the DD of the carrier phase measurements $\boldsymbol{\nabla\Delta L_{IF}}$. Further, taking into account Table 1, the estimation of an RMS error of the carrier phase measurements is used to evaluate an RMS error for initial carrier phase measurements (as a component of a corresponding difference) considering an amplification factor.

In particular, according to Table 1, the assessment of an RMS error of a random pseudonoise error of initial carrier phase measurements (as a random stationary process) is about 6 times less than an RMS error of DD of a carrier phase from ionosphere-free linear combinations of initial measurements.

Further, errors of DD of a carrier phase are considered as a residual vector which components are the differences of the mentioned DD of a carrier phase and the optimum polynomial representations [2] corresponding to them as an adequate representation of the measured function. It is supposed that results of measurements and their double differences are presented in groups – sessions of measurements. The session of measurements is an arranged sequence of measurements on the time interval of *Hs* duration in number of not less than 30 [2–4, 6] with an approximately same 10-second (for the equipment of satellite navigation GS SC) pace on time.

The choice of an optimum order of a polynom (optimum polynom) of approximation of the results of measurements on an interval of *Hs* can be carried out by the following criteria:

1. *M* – on control of a methodical error of approximation of calculated values of the measurements calculated in a priori known coordinate and high-speed parameters of the movement (CHSPM) of LEO spacecraft. Generally, CHSPM are calculated by integration of a system of differential equations of the SC movement at the known statistical assessment of initial conditions (IC) – CHSPM on the fixed (initial) timepoint. It is supposed that the mentioned statistical assessment of IC is calculated in the ground control complex (GCC) of LEO spacecraft on trajectory measurements by a method of the smallest squares or is a priori known. An optimum polynom by this criterion is the polynom of the minimum order at which the module of a methodical error of approximation of settlement measurements should not exceed the control size *M (dR)* set in advance;

2. *C* – on control of speed of change of an RMS error of approximation of the results of measurements from a polynom order. An optimum polynom is a polynom of the minimum i-th order at which the distinction of percentage of the module of a difference of an RMS error for adjacent i-th and $(i + 1)$-th polynoms in relation to an RMS error of the i-th polynom does not exceed the set level *C (dR)*.

A polynomial residual is a difference between the initial measurement and the calculated value corresponding to it calculated with the use of an optimum polynom.

A repeated testing of procedures of definition of optimum polynoms with use of real measurements of the equipment of satellite navigation of GS SC has shown a high level of the coincidence of optimum orders of polynoms by criteria of *M* and *C* at *M (dR)* = 12–18 mm and *C (dR)* = 20% and equivalence of the results received by these criteria.

2. An assessment of errors of phase measurements of the equipment of satellite navigation

2.1. Conditions and estimation of an RMS error measurements

As an example, employing the offered technique of an assessment of hardware errors of phase measurements (on a carrier phase) has been carried out in relation to GNSS-measurements of the equipment of satellite navigation of GS SC in the posteriori mode on a specialized automated working place of Joint Stock Company "Russian Space Systems" applying the information files containing results of navigation definitions and navigation (trajectory) information, in particular, carrier phase measurements.

To evaluate hardware errors of phase measurements, the average of statistical estimates of a mean square error of measurements and dispersion of an RMS error on selections were used.

Calculation of an RMS error was performed on measurements of a carrier phase of the equipment of satellite navigation and from the ground IGS measuring stations (in the RINEX files format) on private selections of S1 and S2.

Calculation of a required RMS error measurements of phase carrier as dot estimates was carried out taking into account the normative documents [6] and recommendations stated in [2,3,5]. The following parameters were calculated on each selection:

1. An RMS error for each session of DD of a carrier phase

$$\nabla\delta_j = \sqrt{\frac{\sum_{i=1}^{n}(r_i - r_p)^2}{n - N}}$$

where r_i and r_p is the double difference from compiling initial simultaneous carrier phase measurements and a calculated value corresponding to this difference, respectively, n is the number of double differences in the session, N is the order of an optimum polynomial, $j = 1,2,..,k$ is the serial number of the session of second differences, k is the general number of sessions;

2. An average RMS error on all sessions of DD of a carrier phase:

$$\nabla m_s = \frac{\sum_{j=1}^{k} \nabla\delta_j}{k}.$$

3. Hence, dispersion and the RMS error for ∇m_s is:

$$D_s = \frac{\sum_{j=1}^{k}(\delta_j - \nabla m_s)^2}{k-1},$$

$$\nabla\delta_s = \sqrt{\nabla D_s};$$

4. A confidence span for DD of a carrier phase is:

$$\nabla L = (\nabla a_1, \nabla a_2)$$

where $\nabla a_1 = \nabla m_s - t_\beta \nabla\delta_s, \nabla a_2 = \nabla m_s + t_\beta \nabla\delta_s$ determine the limits of the ∇L interval, where with the p probability there is a true value for ∇m_s is equal with the probability $\beta=1-p$ – outside this interval, t_β defines the shift of the limits of the interval from its center in the units of an RMS error for the chosen p (according to the Student's distribution). Further, it is accepted that $p = 0.9973$, hence $\beta=1-p=0.0027$. At $p = 0.9973$, $t_\beta = 3$.

5. The evaluation of the finding of ∇m_s outside the limits of a confidence span is

$$\beta^* = \frac{k_n}{k}$$

where k_n is the number of sessions with m_s outside a confidence span.

6. An RMS error of the carrier phase measurements considering an amplification factor:

$$= \delta_s / k_i, \ i = 1.2.$$

2.2 The results of the evaluation of the errors of the phase measurements of the equipment of satellite navigation

The tables (1, 2) given below give the results of statistical valuations of the errors of carrier phase measurements obtained using DD of a carrier phase of the equipment of satellite navigation for the baseline AJAC-GSC1 (AJAC is a ground measuring station IGS, GSC1 is the equipment of satellite navigation of GS SC).

To determine initial statistical evaluations, there were used two groups made of 35 and 36 sessions of DD of a carrier phase in the S1 and S2 selections, respectively (in each session not less than 30 measurements). The two groups of sessions are presented in Figs. 1, 2 in the form of graphs of residuals of the mentioned DD of a carrier phase from the optimum polynomial representation (in mm).

Tables 1 and 2 use the following symbols:

No. is the serial number of the session of DD of a carrier phase;

RXX is the number of the working point of a navigation spacecraft of GLONASS XX;

k_1, k_2 is the minimum and maximum values, respectively, of an amplification factor of the RMS error of DD of a carrier phase in relation to the RMS error of initial measurements of a carrier phase, $k_1 = \nabla\Delta\varepsilon_{IF}$ (at $\varepsilon_1 = \varepsilon_2 = 5.917$), $k_2 = \nabla\Delta\varepsilon_{IF}$ (at $\gamma = \varepsilon_2/\varepsilon_1 = 6.413$);

∇L, ∇L_i is the confidence span, respectively, for DD of a carrier phase and initial measurements of a carrier phase at $t_\beta = 3$ (t_β defines the number of corresponding RMS errors, which should be decreased and increased by ∇m_s) to make a true value of ∇m_s present in the confidence span with the probability of $\beta = 0.9973$).

The statistical evaluations of the results of processing of the sessions of DD of a carrier phase of the S1 selection:

1. An average RMS error on the total sessions of DD: $\nabla m_s = 8.56$ mm.

2. An RMS error for ∇m_s (for an average RMS error of all sessions): $\nabla\delta_s = 1.88$ mm.

3. A confidence span for DD: $\nabla L = (2.92, 14.20)$ mm.

4. A confidence span for initial phase measurements of pseudorange (calculated by the ∇L value considering amplification factors):

$$\nabla L_i = (2.92/k_i, 14.20/k_i) \text{ mm},$$

$$\nabla L_1 = (0.49, 2.40) \text{ mm},$$

$$\nabla L_2 = (0.46, 2.21) \text{ mm}.$$

5. β^* is the evaluation of the presence frequency of ∇m_s (an average RMS error) beyond the limits of a confidence span: $\beta^* = 0$.

6. Average RMS errors of phase measurements of pseudorange (calculated by the ∇m_s value considering amplification factors k1, k2): $\nabla m_{sk1} = 1.45$ mm, $\nabla m_{sk2} = 1.33$ mm.

7. An RMS error for ∇m_{ski} of phase measurements considering an amplification factor (calculated by the $\nabla\delta_s$ value considering the amplification factors k1, k2): $\delta_{sk_1} = 0.32$ mm, $\delta_{sk_2} = 0.29$ mm.

The statistical evaluations of the results of processing of the sessions of DD of a carrier phase of the S2 selection:

Table 2. Statistical assessments of measurement errors of a carrier phase of the equipment of satellite navigation of GS SC of the S1 residual.

No.	Navigation SC1 – Navigation SC 2	Session duration, s	Quantity of measurements in the session	$\nabla\delta$ – an RMS error (mm) of residuals of DD of a carrier phase	$\nabla\delta_{k1}$ – an RMS error (mm) of initial measurements of a carrier phase, k1 = 5.917	$\nabla\delta_{k2}$ – an RMS error (mm) of initial measurements of a carrier phase, k2 = 6.413
1	R20 – R21	379.999000	39	8.592000	1.452087	1.339779
2	R11 – R22	360.000000	37	7.303000	1.234240	1.138781
3	R09 – R16	869.999000	88	9.885000	1.670610	1.541400
4	R19 – R10	709.999000	72	7.247000	1.224776	1.130048
5	R10 – R01	710.001000	72	9.784000	1.653541	1.525651
6	R10 – R20	580.001000	59	8.114000	1.371303	1.265242
7	R21 – R22	380.000000	39	6.390000	1.079939	0.996414
8	R10 – R11	459.999000	47	6.049000	1.022309	0.943240
9	R10 – R20	380.000000	39	7.556000	1.276998	1.178232
10	R16 – R10	420.000000	43	9.577000	1.618557	1.493373
11	R10 – R20	379.999000	39	9.938000	1.679567	1.549665
12	R10 – R20	420.000000	43	8.594000	1.452425	1.340090
13	R11 – R21	330.000000	34	7.004000	1.183708	1.092157
14	R19 – R09	870.000000	88	8.922000	1.507859	1.391237
15	R19 – R09	579.999000	59	7.340000	1.240493	1.144550
16	R11 – R21	320.001000	33	5.698000	0.962988	0.888508
17	R19 – R10	289.999000	30	9.338000	1.578165	1.456105
18	R11 – R22	459.999000	47	6.545000	1.106135	1.020583
19	R10 – R20	320.000000	33	10.458000	1.767450	1.630750
20	R20 – R21	789.998000	80	12.370000	2.090586	1.928894
21	R18 – R19	489.998000	50	8.744000	1.477776	1.363480
22	R01 – R11	899.999000	91	11.669000	1.972114	1.819585
23	R21 – R22	589.999000	60	5.710000	0.965016	0.890379
24	R20 – R11	519.999000	53	7.874000	1.330742	1.227818
25	R09 – R10	459.999000	47	6.557000	1.108163	1.022454
26	R16 – R20	330.000000	34	10.222000	1.727565	1.593950
27	R11 - R21	379.999000	39	6.508000	1.099882	1.014814
28	R20 – R11	320.000000	33	11.749000	1.985635	1.832060
29	R20 – R01	580.001000	59	9.681000	1.636133	1.509590
30	R22 – R05	470.001000	48	9.160000	1.548082	1.428349
31	R09 – R21	519.999000	53	7.626000	1.288829	1.189147
32	R11 – R21	380.001000	39	7.618000	1.287477	1.187900
33	R08 – R10	579.999000	59	9.241000	1.561771	1.440979
34	R20 – R11	380.000000	39	10.842000	1.832347	1.690628
35	R20 – R11	300.000000	31	6.085000	1.028393	0.948854
36	R09 – R08	580.000000	59	12.090000	2.043265	1.885233
37	All sessions	494.166306	50	8.557778	1.446303	1.334442

Table 3. Statistical assessments of measurement errors of a carrier phase of the equipment of satellite navigation of GS SC of the S2 residual.

No.	Navigation SC1 – Navigation SC 2	Session duration, s	Quantity of measurements in the session	$\nabla\delta$ an RMS error (mm) of residuals of DD of a carrier phase	$\nabla\delta_{k1}$ – an RMS error (mm) of initial measurements of a carrier phase, k1 = 5.917	$\nabla\delta_{k2}$ – an RMS error (mm) of initial measurements of a carrier phase, k2 = 6.413
1	R09 – R20	539.999000	55	12.544000	2.119993	1.956027
2	R01 – R09	540.000000	55	8.341000	1.409667	1.300639
3	R21 – R02	360.000000	37	13.431000	2.269900	2.094340
4	R09 – R21	319.999000	33	5.312000	0.897752	0.828317
5	R11 – R21	400.000000	41	4.703000	0.794828	0.733354
6	R07 – R09	409.999000	42	9.624000	1.626500	1.500702
7	R20 – R11	719.999000	73	8.027000	1.356600	1.251676
8	R10 – R20	699.999000	71	6.963000	1.176779	1.085763
9	R09 – R20	440.000000	45	8.115000	1.371472	1.265398
10	R11 – R22	419.999000	43	7.549000	1.275815	1.177140
11	R08 – R10	710.001000	72	7.339000	1.240324	1.144394
12	R10 – R01	540.001000	55	7.865000	1.329221	1.226415
13	R20 – R19	440.001000	45	6.473000	1.093967	1.009356
14	R11 – R21	360.000000	37	9.088000	1.535913	1.417121
15	R19 – R11	539.999000	55	7.619000	1.287646	1.188056
16	R11 – R21	370.000000	38	6.008000	1.015379	0.936847
17	R19 – R11	719.999000	73	13.280000	2.244381	2.070794
18	R08 – R10	539.999000	55	13.180000	2.227480	2.055200
19	R11 – R21	720.000000	73	12.872000	2.175427	2.007173
20	R10 – R01	740.000000	75	7.925000	1.339361	1.235771
21	R21 – R22	720.001000	73	11.145000	1.883556	1.737876
22	R21 – R22	329.999000	34	6.997000	1.182525	1.091065
23	R20 – R19	320.000000	33	9.477000	1.601656	1.477780
24	R10 – R09	430.000000	44	10.201000	1.724016	1.590675
25	R20 – R19	540.001000	55	14.275000	2.412540	2.225947
26	R01 – R19	719.999000	73	16.344000	2.762211	2.548573
27	R10 – R20	400.000000	41	7.487000	1.265337	1.167472
28	R20 – R11	519.999000	53	7.937000	1.341389	1.237642
29	R20 – R11	399.999000	41	8.999000	1.520872	1.403243
30	R21 – R02	400.000000	41	7.357000	1.243367	1.147201
31	R10 – R01	440.001000	45	12.836000	2.169343	2.001559
32	R21 – R22	369.999000	38	5.281000	0.892513	0.823484
33	R10 – R11	329.999000	34	6.044000	1.021464	0.942461
34	R09 – R20	339.999000	35	10.275000	1.736522	1.602214
35	R10 – R11	419.999000	43	8.348000	1.410850	1.301731
36	All sessions	491.713971	50	9.121743	1.541616	1.422383

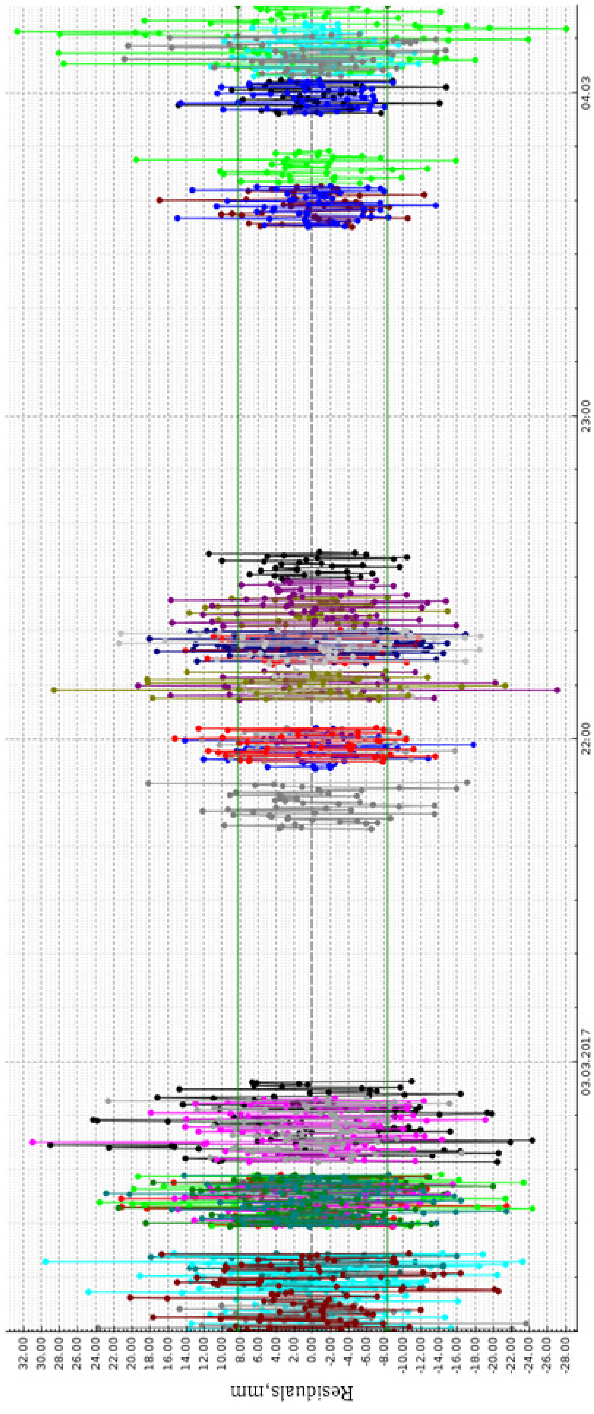

Fig. 1. A graph of residuals of DD of a carrier phase from an optimum polynomial representation of measurement sessions of the S1 selection

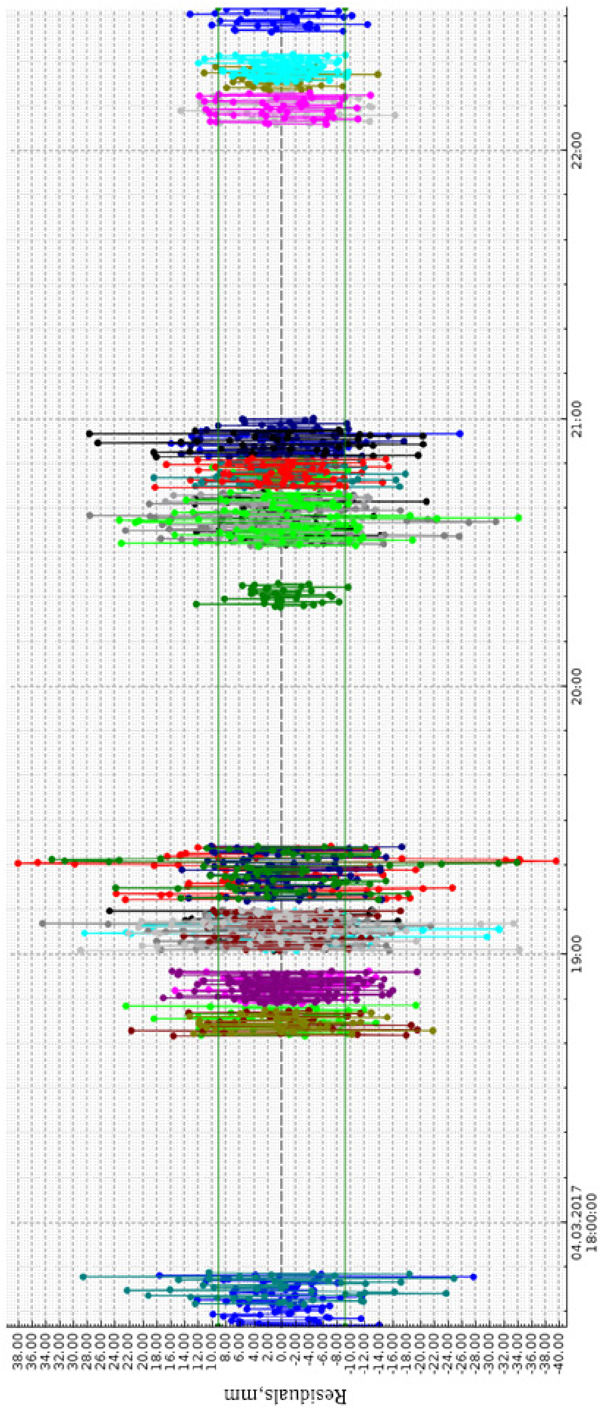

Fig. 2. A graph of residuals of DD of a carrier phase from an optimum polynomial representation of measurement sessions of the S2 selection

1. An average RMS error on total sessions of DD: ∇m_s = 9.12 mm.

2. An RMS error for ∇m_s (for an average RMS error of all sessions): $\nabla \delta_s$ = 2.89 mm.

3. A confidence span for DD: ∇L = (0.46,17.78) mm.

4. A confidence span for initial phase measurements of pseudorange (calculated by the ∇L value considering amplification factors):

∇L_i = (0.46 / k_i ,17.78 / k_i) mm, ∇L_1 = (0.08,3.00) mm, ∇L_2 = (0.07,2.77) mm.

5. β^* is the evaluation of the frequency of presence of ∇m_s (an average RMS error) beyond the limits of a confidence span: β^* = 0.

6. Average RMS errors for initial phase measurements of pseudorange (calculated by the ∇m_s value considering the amplification factors k_1, k_2): ∇m_{sk1} = 1.54 mm, ∇m_{sk2} = 1.42 mm.

7. An RMS error for ∇m_{ski} of phase measurements considering amplification factor (calculated by $\nabla \delta_s$ value considering the amplification factors k_1, k_2): δ_{sk_1} = 0.49 mm, δ_{sk_2} = 0.45 mm.

According to the offered technique, an evaluation of the precise characteristics of measurements of pseudoranges on the phase of the carrier phase received from the equipment of the satellite navigation of GS SC was carried out. The results of the evaluations are calculated on two selections having 36 (the S1 selection) and 35 (the S2 selection) sessions, respectively, of DD with at least lot less than 30 measurements in each session.

The final results of the mentioned evaluations are given in Table 4.

Table 4. The evaluations of the precise characteristics of the carrier phase characteristics

Selection	max ∇m_{ski}, mm	∇L_l, мм	∇L_r, mm	β^*
S1	1.45	0.46	2.40	0
S2	1.54	0.07	3.00	0

Table 4 uses the following symbols:

max∇m_{ski} is the maximum value of an RMS error of initial measurements of a carrier phase for the group at the amplification factors k_i, i=1.2;

∇L is the confidence span of an average of a statistical evaluation of an RMS error;

p is the possibility of finding of an average of the evaluation of an RMS error in ∇L;

β^* is the assessment of the possibility of the finding of ∇m_s beyond ∇L;

∇L_l is the left limit of a confidence span ∇L (a minimum limit of ∇L at the amplification factors k_i, i=1.2 with the possibility of confidence p = 0.9973);

∇L_r is the right limit of the confidence span of ∇L (a minimum limit of ∇L at the amplification factors ki, i=1.2 with the possibility of confidence p = 0.9973 is equal to “nonconfidence” – β = 1- 0.9973= 0.0027).

The obtained statistical characteristics in the given Tables 1–3 and the graphs in the Figs. 1–2 are the representation evaluations of hardware errors of measurements on a carrier phase for the chosen spacecraft.

Conclusion

1. The technique to evaluate a hardware error of navigation measurements of a carrier phase received in a two-frequency mode by onboard navigation systems of LEO spacecraft is offered.

2. Selective evaluations of hardware errors of navigation measurements of a carrier phase received by the equipment of satellite navigation of GS SC are received. In particular, it was determined that with the probability of p = 0.9973, an average value of an RMS error of measurements of a carrier phase using a navigation spacecraft GLONASS is in the range 2–3 mm, that agrees with the RMS error for other similar instruments.

3. The offered technique and technology of the evaluation of hardware errors of the carrier phase measurements received in the two-frequency mode of the equipment of satellite navigation of LEO GS SC can be used as typical of monitoring precisions of GNSS-measurements of a wide class of Russian and foreign LEO spacecraft.

4. A visual analysis of residuals of DD of a carrier phase given in Figs. 1,2 allows one to make an assumption that there is a small trend in them, the presence of which can be explained by the influence of the signal propagation medium. The conclusion can be obtained based on the results of processing the measurements on long intervals of time and is an item of further research.

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